ALGEBRA

SECOND TERM

PREPARATORY TWO

Lesson (1)

Factorizing quadratic Trinomial

Factorize each of the following by taking out the highest common factor :

1. $3x + 21y = 3\,(\cdots\cdots + \cdots\cdots)$
2. $2a^3 + 6a^2 - 4a = \cdots\cdots\,(\cdots\cdots + \cdots\cdots - \cdots\cdots)$
3. $3x^2 + 15xz + 21xy^2 = \cdots\cdots\,(\cdots\cdots\cdots\cdots\cdots\cdots)$
4. $(x-5)\,x^2 + (x-5)\,y^2 = \cdots\cdots\cdots\cdots\cdots\cdots$
5. $a\,(a-b) - b\,(b-a) = \cdots\cdots\cdots\cdots\cdots\cdots$

Factorize each of the following :

1. $x^2 + 5x + 6$
2. $x^2 - 5x + 6$
3. $x^2 + 5x - 6$
4. $x^2 - 5x - 6$

Complete each of the following :

1.	Two numbers such that their product = 30 and their sum is 11
2.	Two numbers such that their product = 12 and their sum is – 8
3.	$x^2 - 11x + 18 = (x - \cdots\cdots)\,(x - \cdots\cdots)$
4.	$x^2 + 5x + 6 = (\cdots\cdots)\,(x + 2)$
5.	$(x - \cdots\cdots)$ is a factor of the expression $x^2 - x - 6$
6.	If $(x + 2y) = 4$ and $(x - y) = 1$, then the numerical value of the expression $x^2 + xy - 2y^2$ is $\cdots\cdots$
7.	If $k \in \mathbb{Z}$, $x^2 + kx - 3$ can be factorized , then $k = \cdots\cdots$
8.	The rectangle whose area is $(x^2 - 7x + 6)$ square unit and if its length is $(x - 6)$ length unit , then its width is $\cdots\cdots$ length unit.

Homework

1.	Two numbers such that their product = – 18 and their sum is 3
2.	Two numbers such that their product = – 15 and their sum = – 14
3.	$x^2 + \cdots\cdots + 35 = (x + \cdots\cdots)\,(\cdots\cdots + 5)$
4.	If $(x - 2)$ is a factor of the expression $x^2 - 8x + 12$, then the other factor is $\cdots\cdots$

5. If $(a - b) = 1$ and $(x + y) = -3$, then $a(x + y) - b(x + y) =$ ....

6. If $(x - 4)$ is a factor of the expression $x^2 - 5x + 4$, then the other factor is ....

7. $x^2 +$ .... $+ 8 = (x + 2)($ .... $+$ .... $)$

Choose the correct answer :

1. If $a - b = 3$, then $6a - 6b =$ ....
(a) 2 (b) 9 (c) 18 (d) 3

2. The expression : $x^2 - x - a$ can be factorized if $a =$ ....
(a) 3 (b) 4 (c) 5 (d) 6

3. If $x^2 - 2x - k = (x + 3)(x - 5)$, then $k =$ ....
(a) -2 (b) -8 (c) 15 (d) 2

4. The expression $x^2 - 3x + c$ can be factorized when $c =$ ....
(a) 1 (b) 2 (c) 4 (d) 6

5. If the expression $x^2 + bx - 10$ can be factorized , then b may be ....
(a) 3 (b) 2 (c) 1 (d) -1

6. The number which can be added to the expression : $x^2 - 11x + 15$ to be factorized is ....
(a) 1 (b) 2 (c) 3 (d) 4

Homework

1. If $x - y = 3$, $x - 2y = 5$, then $x^2 - 3xy + 2y^2 =$ ....
(a) 15 (b) 8 (c) 2 (d) -2

2. The expression $x^2 + 7x + a$ can be factorized if $a =$ ....
(a) 8 (b) 10 (c) 18 (d) 49

3. For the expression $x^2 - x - k$ can be factorized then $k \neq$ ....
(a) 12 (b) 30 (c) 6 (d) 8

4. If the expression $x^2 + ax + 2$ can be factorized , then a may be ....
(a) 1 (b) 2 (c) 3 (d) 4

5. If the expression $x^2 - c\,x + 12$ can be factorized , then c may be ....

(a) -1 (b) 4 (c) 7 (d) 1

6. The number which can be added to the expression $x^2 - 8\,x + 5$ to be factorized is ....

(a) 1 (b) 2 (c) 4 (d) 5

Factorize each of the following perfectly:

1. $x^2 + 8\,x + 15$
2. $x^2 - 17\,x + 30$
3. $x^2 - 6\,x - 16$
4. $b^2 + 3\,bc - 10\,c^2$
5. $x^2 - 7\,x\,y - 18\,y^2$
6. $15\,a + a^2 - 34$
7. $x^2 + 21 - 10\,x$
8. $x\,(x + 7) + 10$
9. $(x - 1)^2 - 2\,(x - 1) - 8$

....

Homework .

1. $x^2 + 11\,x + 10$
2. $x^2 - 7\,x + 12$
3. $x^2 + 5\,x - 14$
4. $x^2 + 4\,x - 12$
5. $x^2 - 3\,x - 10$
6. $\ell^6 - 6\,\ell^3 - 40$
7. $x^2 - 4\,x - 3\,(x - 2)$

....

Lesson (2)

Factorizing quadratic Trinomial Follow

Complete each of the following :

1. $5y^2 + 16y + 3 = (5y + \dots\dots)(y + \dots\dots)$
2. $5x^2 - 2x - 7 = (5x - \dots\dots)(x + \dots\dots)$
3. $3x^2 + 10x + 8 = (\dots\dots + 4)(x + \dots\dots)$
4. $6x^2 - 11x - 10 = (2x - \dots\dots)(\dots\dots + 2)$
5. $3x^2 + 7x - 6 = (3x - \dots\dots)(\dots\dots + \dots\dots)$
6. If $9x^2 + 39x + 36 = 3(3x + c)(x + 3)$, then $c = \dots\dots$
7. If $(x + 1)$ is a factor of the expression $2x^2 - x - 3$, then the other factor is ....
8. If $a(x + y) - b(x + y) = 15$ and $a - b = 3$, then $x + y = \dots\dots$
9. If $a^2 + k + 6 = (a - 3)(a - 2)$, then $k = \dots\dots$

Homework

1. $2x^2 + x - 6 = (\dots\dots - \dots\dots)(x + \dots\dots)$
2. $2x^2 - \dots\dots - \dots\dots = (2x + 3y)(\dots\dots - 2y)$
3. $5x^2 - 3xy - \dots\dots = (x - y)(\dots\dots + \dots\dots)$
4. $3a^2 - 5a - 2 = (3a + \dots\dots)(a - \dots\dots)$
5. If $x + 3y = 7$, $x - y = 3$, then $x^2 + 2xy - 3y^2 = \dots\dots$

Choose the correct answer :

1. If $x^2 + ax - 13 = (x + 1)(x - 13)$, then $a = \dots\dots$

   (a) zero (b) 25 (c) -12 (d) 12

2. $x^2 + 7x + c$ can be factorized if c = ………

(a) 12 (b) – 12 (c) 17 (d) 9

3. The number which can be added to the expression : $2x^2 + 5x - 10$ to be factorized is ………

(a) – 1 (b) – 2 (c) – 3 (d) – 4

Homework

1. If $2x^2 - cx - 3 = (2x - 1)(x + 3)$, then c = ………

(a) 5 (b) – 5 (c) 7 (d) – 7

2. $6x^2 - 7x - 3 =$ ………

(a) $(3x - 1)(2x - 3)$ (b) $(3x + 1)(2x - 3)$

(c) $(3x + 1)(2x + 3)$ (d) $(3x - 1)(2x + 3)$

3. The rectangle whose area is $(2x^2 - 3x - 5)$ cm² and one of its dimensions is $(x + 1)$ cm. , the second dimension is ……… cm.

(a) $(x - 5)$ (b) $(2x - 5)$ (c) $(2x + 5)$ (d) $(2x - 3)$

Factorize each of the following perfectly:

1. $2x^2 + 3x + 1$
2. $5z^2 - 7z + 2$
3. $3x^2 - 14x - 5$
4. $3x^2 + 10x + 8$
5. $8z^2 + 2z - 3$
6. $3x^2 - 20xy - 7y^2$
7. $21x^2y^2 + 6x^2y^3 - 15x^2y^4$

Homework

1. $3a^2 + 7a + 2$

2.	$3x^2 - 10x + 7$
3.	$5x^2 + 4x - 12$
4.	$8x^2 + 14x + 5$
5.	$6x^2 - 11x + 3$
6.	$3y^2 + 7y - 6$
7.	$25m - 10 + 15m^2$ ....

Lesson (3)

Factorizing a perfect square trinomial

The perfect square trinomial has the following properties :

1. The first term is a perfect square and it is always positive.
2. The third term is a perfect square and it is positive also.
3. The middle term $= \pm 2\sqrt{1^{st}\text{term}} \times \sqrt{3^{rd}\text{term}}$

If the trinomial is a perfect square , then :

1. The middle term $= \pm 2 \times \sqrt{\text{the first term}} \times \sqrt{\text{the third term}}$
2. The first term $= \dfrac{(\text{the middle term})^2}{4 \times \text{the third term}}$
3. The third term $= \dfrac{(\text{the middle term})^2}{4 \times \text{the first term}}$

If the trinomial is a perfect square , then we can factorize it to be in the form :

$(\sqrt{\text{The first term}} \pm \sqrt{\text{The third term}})^2$

Complete to get a perfect square:

1.	$4x^2$ .... $+ 1$
2.	$9a^2$ .... $+ 36$

3.	$\frac{1}{25}x^2$ ………. + $\frac{1}{4}y^2$
4.	………. $- 18y^2 + 81$
5.	The value of m which makes the expression : $4x^2 + 12x + m$, a perfect square is ……….

Homework .

1.	$4a^2$ ………. $+ 36b^2$
2.	z^4 ………. $+ 49\ell^2$
3.	$25m^2 + 20mn +$ ……….

Choose the correct answer :

1. If $x^2 + kx + 16$ is a perfect square , then k = ……….
   (a) 4 (b) ± 4 (c) ± 8 (d) 1

2. If $x^2 - 2xy + y^2 = 25$, then $x - y =$ ……….
   (a) 25 (b) – 5 (c) 5 (d) ± 5

3. $5x^2 - 8xy - 4y^2 =$ ……….
   (a) $(5x + 2y)(x - 2y)$ (b) $(5x - 2y)(x + 2y)$
   (c) $(5x - 4y)(x + y)$ (d) $(x - 4y)(5x + y)$

4. If $a^2 + b^2 = 11$, $ab = 5$, then $a - b =$ ……….
   (a) 6 (b) ± 1 (c) 1 (d) – 1

5. The value of c which makes the expression $cx^2 + 10x + 1$ a perfect square is ……….
   (a) 25 (b) 10 (c) 9 (d) 5

6. If $x = 6$, $y = 4$, then $x^2 - 2xy + y^2 =$ ……….
   (a) 2 (b) 4 (c) 10 (d) 100

Homework

1. The expression : $a x^2 - 40x + 25$ is a perfect square when a = ...
(a) 2 (b) 4 (c) 9 (d) 16

2. If $x^2 + kx + 25$ is a perfect square, then k = ...
(a) 5 (b) 10 (c) ± 10 (d) ± 5

3. If the expression $x^2 + ax + 16$ is a perfect square, then a = ...
(a) zero (b) ± 16 (c) ± 4 (d) ± 8

4. If the expression $x^2 + 14x + b$ is a perfect square, then b = ...
(a) 2 (b) 7 (c) 14 (d) 49

5. The value of k which makes the expression $16x^2 - 24x + k$ a perfect square is ....
(a) 6 (b) 9 (c) 12 (d) 24

6. The expression $a x^2 - 40x + 25$ is a perfect square when a = ...
(a) 2 (b) 4 (c) 9 (d) 16

7. If the expression $c + 3x + \frac{1}{4}$ is a perfect square, then c = ...
(a) 9 (b) $\frac{9}{4}x^2$ (c) $9x^2$ (d) $4x^2$

Factorize each of the following perfectly:

1. $m^2 - 2m + 1$
2. $x^2 + 2xy + y^2$
3. $9a^2 + 6ab + b^2$
4. $36 - 60k + 25k^2$
5. $6a^4 - 12a^2b^2 + 6b^4$
6. $24x + 24x^2 + 6x^3$

....

7. $4b^2c + bc^2 + 4b^3$

....

8. $(c - d) + 2x(c - d) + x^2(c - d)$

....

9. $\frac{1}{16}a^2 + \frac{1}{10}a + \frac{1}{25}$

10. $9a^2 + 5b(5b - 6a)$

11. $4x^2 - 7y(4x - 7y)$

Homework

1. $9x^2 + 12x + 4$

2. $25b^2 - 10b + 1$

3. $4x^2 - 4xy + y^2$

4. $18y^2 - 12y + 2$

....

5. $20ay^2 - 60ay + 45a$

....

6. $3z + 42z^4 + 147z^7$

....

7. $0.01x^2 - 0.2x + 1$

Use the factorization to find the value of each:

1. $(87)^2 + 2 \times 13 \times 87 + (13)^2$

....

2.	$(997)^2 + 6 \times 997 + 9$ ....

Homework

1.	$(7.3)^2 + 2 \times 7.3 \times 2.7 + (2.7)^2$ ....
2.	$(20.7)^2 - 1.4 \times 20.7 + (0.7)^2$ ....

Lesson (4)

Factorizing the difference of two squares

The difference of two squares of two quantities
= (the sum of the two quantities) × (the difference of the two quantities)

$$\boxed{a^2 - b^2 = (a + b)\,(a - b)}$$

Complete each of the following :

1.	$(2x + \cdots\cdots)\,(\cdots\cdots - 3y) = 4x^2 - \cdots\cdots$
2.	$(\cdots\cdots + 3m)\,(\cdots\cdots - 3m) = 25x^2 - \cdots\cdots$
3.	If $a - b = 2$, $a + b = 3$, then $a^2 - b^2 = \cdots\cdots$
4.	If $x^2 - y^2 = 20$, $x + y = 10$, then $x - y = \cdots\cdots$
5.	If $x^2 - y^2 = x + y$, then $x - y = \cdots\cdots$
6.	If $a + b = 7\,(a - b) = 14$, then $a^2 - b^2 = \cdots\cdots$
7.	If $(39)^2 - 1 = 40x$, then $x = \cdots\cdots$
8.	$\frac{1}{2}x^2 - 2 = \frac{1}{2}(\cdots\cdots)(\cdots\cdots)$

Homework

1. $\cdots\cdots - 64x^2 = (4 - \cdots\cdots)(4 + \cdots\cdots)$
2. If $a^2 - b^2 = 45$, $a - b = 5$, then $\sqrt{a + b} = \cdots\cdots$
3. If $2(a - b)(a + b) = 18$, then $a^2 - b^2 = \cdots\cdots$
4. If $x + y = 5$, $x - y = 1$, then $x^2 - y^2 = \cdots\cdots$
5. If $x + y = 3(x - y) = 12$, then $x^2 - y^2 = \cdots\cdots$
6. $(75)^2 - (25)^2 = 100 \times \cdots\cdots$
7. $3x^2 - 5x - 2 = (3x + \cdots\cdots)(\cdots\cdots - 2)$

Choose the correct answer :

1. If $x^2 - a = (x - 3)(x + 3)$, then $a = \cdots\cdots$
   (a) 3 (b) -3 (c) 9 (d) -9
2. If $x^2 + \ell - 4 = (x - 2)(x + 2)$, then $\ell = \cdots\cdots$
   (a) zero (b) 2 (c) 4 (d) 8
3. If $a - b = 7$, $a + b = 5$, then $2a^2 - 2b^2 = \cdots\cdots$
   (a) 2 (b) 12 (c) 35 (d) 70
4. 📖 If $x^2 - y^2 = 16$, $y - x = 2$, then $x + y = \cdots\cdots$
   (a) 4 (b) 8 (c) -8 (d) 2
5. If $(25)^2 - (15)^2 = 10x$, then $x = \cdots\cdots$
   (a) 40 (b) 30 (c) 20 (d) 10
6. $(x - y)(x + y)(x^4 - 2x^2y^2 + y^4) = \cdots\cdots$
   (a) $x^6 - y^6$ (b) $(x - y)^3(x + y)^3$
   (c) $(x^3 - y^3)(x^3 + y^3)$ (d) $(x^2 + y^2)(x^2 - y^2)$
7. If $a + b = 8$, $b - a = -5$, then $a^2 - b^2 = \cdots\cdots$
   (a) -40 (b) 40 (c) 13 (d) -13

Homework

1. If $x + 2y = 3$, $x^2 - 4y^2 = 21$, then $x - 2y = \cdots\cdots\cdots$

(a) 14 (b) 9 (c) 7 (d) 6

2. If $x^2 - y^2 = 24$, $x + y = 8$, then $3x - 3y = \cdots\cdots\cdots$

(a) $\frac{1}{3}$ (b) 3 (c) 9 (d) 16

3. If $a + b = 5$, $a - b = 4$, then $b^2 - a^2 = \cdots\cdots\cdots$

(a) -20 (b) -1 (c) 9 (d) 20

4. If $x^2 - y^2 = 16$, $x + y = 8$, then $x - y = \cdots\cdots\cdots$

(a) 2 (b) 1 (c) 128 (d) 64

5. If the expression : $x^2 + 7x + k$ can be factorized , then $k = \cdots\cdots\cdots$

(a) 16 (b) -12 (c) 30 (d) 6

6. The expression : $4x^2 + k + 25y^2$ is a perfect square when $k = \cdots\cdots\cdots$

(a) 20 (b) $10xy$ (c) $20xy$ (d) $\pm 20xy$

7. $x^2 - \cdots\cdots\cdots = (x - 7)(x + 7)$

(a) 7 (b) 49 (c) -49 (d) -7

8. If $x^2 + 2xy + y^2 = 9$, then $x + y = \cdots\cdots\cdots$

(a) 9 (b) 3 (c) ± 3 (d) ± 9

Factorize each of the following perfectly:

1. $x^2 - 4$
2. $225x^2 - y^2$
3. $625a^2 - 81b^2$
4. $9 - y^2$
5. $a^2 - b^2c^4$

6.	$\frac{1}{9}y^2 - 2\frac{1}{4}$
7.	$0.04x^2 - 0.25y^2$
8.	$x^4 - 16y^4$
9.	$8x^2 - 50$
10.	$27x^3 - 48xy^6$
11.	$\frac{1}{2}x^2 - \frac{1}{18}y^2$
12.	$3x^2 - \frac{3}{16}$
13.	$(a+b)^2 - 4$

Homework

1.	$(x+3)^2 - 25$
2.	$a^2 - 25$
3.	$-9x^2 + 25$
4.	$x^2 - \frac{1}{16}$
5.	$\frac{a^2}{25} - \frac{4b^2}{49}$
6.	$x^4 - 1$
7.	$\frac{1}{3}x^2 - 3$

Use the factorization to find the value of each:

1.	$(77)^2 - (23)^2$
2.	$(75)^2 - (25)^2$
3.	$(95)^2 - 25$

Homework

1.	$(78)^2 - (77)^2$
2.	$(999)^2 - 1$

Lesson (5)
Factorizing the sum and the difference of two cubes

The sum of two cubes of two quantities =

(the first + the second) (the square of the first – the first × the second + the square of the second)

i.e. $a^3 \oplus b^3 = (a + b)(a^2 \ominus ab + b^2)$

The difference between two cubes of two quantities =

(the first – the second) (the square of the first + the first × the second + the square of the second)

i.e. $a^3 \ominus b^3 = (a - b)(a^2 \oplus ab + b^2)$

Complete each of the following :

1.	$x^3 - 1 = (x - 1)$ (……………….)
2.	$x^{12} + y^{15} =$ (……… + ………) (……… – ……… + ………)
3.	If $x - 3$ is a factor of the expression $x^3 - 27$, then the second factor is ………
4.	If $x + y = 2$, $x^2 - xy + y^2 = 8$, then $x^3 + y^3 =$ ………
5.	If $(a + b)^2 = 16$, $a^2 + b^2 = 8$, then $2ab =$ ……… ………………………………………………………………………… …………………………………………………………………………

Homework

1.	$8a^3 + 125 =$ (……… + ………) ($4a^2 - 10a +$ ………)
2.	$8a^3 -$ ……… = (……… – ………) (……… + ……… + 9)
3.	If $4a^2 - 2a + 1$ is a factor of the expression $8a^3 + 1$, then the other factor is ………

4.	$2x^2 - 7x - 15 = (2x + 3)(\cdots\cdots\cdots)$
5.	If $k x^2 + 4x + 1$ is a perfect square , then $k = \cdots\cdots\cdots$
6.	If $(x + 2)$ is a factor of the expression : $x^2 - x - 6$, then the other factor is $\cdots\cdots\cdots$
7.	If $x^2 + a x + 5$ can be factorized , then $a = \cdots\cdots\cdots$

Choose the correct answer :

1. If $x^3 - y^3 = 14$, $x^2 + x y + y^2 = 7$, then $x - y = \cdots\cdots\cdots$

(a) 2 (b) 7 (c) 14 (d) -2

2. If $y^3 - a = (y - 2)(y^2 + 2y + 4)$, then $a = \cdots\cdots\cdots$

(a) 2 (b) 4 (c) 8 (d) -8

3. If $x^3 + 27 = (x + 3)(x^2 + k + 9)$, then "k" equals $\cdots\cdots\cdots$

(a) $-6x$ (b) $-3x$ (c) $3x$ (d) $6x$

4. $(x - y)(x + y)(x^4 + x^2 y^2 + y^4) = \cdots\cdots\cdots$

(a) $x^3 - y^3$ (b) $x^3 + y^3$ (c) $x^6 - y^6$ (d) $x^6 + y^6$

5. If $a^2 + b^2 = 11$, $a b = 5$, then $a - b = \cdots\cdots\cdots$

(a) 6 (b) ± 1 (c) 1 (d) -1

6. If $a - b = 5$, then $a^2 - 2ab + b^2 = \cdots\cdots\cdots$

(a) 25 (b) 20 (c) 15 (d) 10

Homework

1. If $x + y = 3$, $x^2 - x y + y^2 = 5$, then $x^3 + y^3 = \cdots\cdots\cdots$

(a) 15 (b) 25 (c) 8 (d) 7

2. If $x^3 + y^3 = 28$, $x + y = 2$, then $x^2 - x y + y^2 = \cdots\cdots\cdots$

(a) 28 (b) 14 (c) 2 (d) 7

3. If $x^3 - 8 = (x + a)(x^2 + 2x + 4)$, then $a = \cdots\cdots\cdots$

(a) 4 (b) -4 (c) 2 (d) -2

4. $x^3 - k^3 = (x - k)(x^2 + 4x + k^2)$, then $k = \cdots\cdots\cdots$

(a) 2 (b) 4 (c) 16 (d) 64

5. $x^3 + 8 = (x + 2)(\cdots\cdots\cdots)$

(a) $x - 2$ (b) $x^2 + 2x + 4$ (c) $x^2 - 4x + 4$ (d) $x^2 - 2x + 4$

6. If $x^3 + 8 = (x + 2)(x^2 + a + 4)$, then a = ………

(a) x (b) $-x$ (c) $-4x$ (d) $-2x$

7. If $x^2 + e - 16 = (x + 4)(x - 4)$, then e = ………

(a) 8^x (b) zero (c) $-8x$ (d) $-4x$

8. $(x^3 + 64) \div (x + 4) = \cdots\cdots\cdots$

(a) $x^2 + 16$ (b) $x^2 - 4x + 16$

(c) $x^2 + 4x + 16$ (d) $x^2 - 4x - 16$

9. $3x^2y + 6xy = \cdots\cdots\cdots (x + 2)$

(a) $3x$ (b) $3xy^2$ (c) x^2y (d) $3xy$

10. $(64)^2 - (36)^2 = \cdots\cdots\cdots$

(a) 100 (b) 28 (c) 2800 (d) 280

Factorize each of the following perfectly:

1. $8x^3 - 125$

2. $m^3 + 64n^3$

3. $\frac{1}{8}a^3 - 8b^3$

…………………………………………………………………………………………

4. $0.027m^3 - n^3$

5. $8x^3 - 343y^6$

6. $16a^3b + 686b^4$

…………………………………………………………………………………………

7. $x^6 - 7x^3 - 8$

…………………………………………………………………………………………

8. If $x^2 - y^2 = 20$, $x - y = 2$, $x^2 - xy + y^2 = 28$
Find the value of $x^3 + y^3$
....

9. **Factorize the following expression perfectly : $(x^3 - 9)(x^3 + 9) + 17$**
....
....

Homework

1. $x^3 + 8$
2. $x^3 - 1$
3. $512x^3 - y^3$
4. $\ell^3 m - 27m^4$
5. $m^6 + 7m^3 - 8$
....
....
....

Lesson (6)
Factorizing by grouping

Factorize each of the following perfectly:

1. $ax + bx + ay + by$
....
....
....

2. $am - an + m - n$
....
....

3. $a^2 + 2ab + b^2 - c^2$

....

....

....

Homework

1. $x y + 5 y + 7 x + 35$

....

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2. $5\ell - 10m - a\ell + 2am$

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3. $9x^2 - 4a^2 + y^2 + 6xy$

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4. $ab x^2 + b x - a x - 1$

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5. $25x^2 - 30x + 9 - 16y^2$

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6. $x^2 - 9a^2 + y^2 + 2xy$

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Lesson (7)
Factorizing by completing the square

The method of factorization by completing the square :

1 We add to the given expression twice the product of the two square roots of the two perfect square terms and subtract it again not to change the main expression.
2 Using the commutative and associative properties , we rewrite the expression after ordering its terms to get the form :

a perfect square trinomial – a perfect square monomial

3 We factorize the resultant expression as a difference between two squares.
4 If it is possible , we should factorize the resultant expressions (resultant factors) in order that the factorization is perfect.

Factorize each of the following perfectly:

1. $x^4 + 4y^4$

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2. $a^4 + 2500b^4$

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3. $8x^4y^2 + 162z^4y^2$

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4. $x^4 + 9x^2 + 81$

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5.

$m^4 - 11\, m^2n^2 + n^4$

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6. $4\,x^4 + 25\,y^4 - 29\,x^2\,y^2$

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Homework

1. $81\,x^4 + 4\,z^4$

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2. $4\,x^4 + 625\,z^4$

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3. $9\,x^4 - 25\,x^2 + 16$

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4. $x^4 + x^2\,y^2 + 25\,y^4$

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5. $16\,x^4 - 28\,x^2\,y^2 + 9\,y^4$

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Lesson (8)

Solving quadratic equations in one variable algebraically

Complete each of the following :

1. If -5 is a root of the equation : $x^2 + 2x - 15 = 0$, then the other root is ....

2. If $x = 2$ is a root of the equation : $x^2 - 6x + k = 0$, then $k =$ .... and the other root is ....

3. If one of the roots of the equation : $2x^2 + 8x = 0$ is a root of the equation : $x^2 + 5x + a = 0$, then $a =$ .... or ....

Homework

1. If the number 9 is a solution of the equation : $x^2 + k = 0$, then $k =$ ....

2. The solution set of the equation : $x^2 + 25 = 0$ in $\mathbb{R}$ is ....

3. The solution set of the equation $x^2 = 4x$ in $\mathbb{R}$ is ....

Choose the correct answer :

1. The S.S. of the equation : $3(x-2)(x+5) = 0$ in $\mathbb{R}$ is ....
(a) $\{0, 2, -5\}$ (b) $\{3, 2, -5\}$ (c) $\{2, -5\}$ (d) $\{-2, 5\}$

2. The S.S. of the equation : $x^2 - 4 = 0$ in $\mathbb{R}$ is ....
(a) $\{4\}$ (b) $\{4, -4\}$ (c) $\{2\}$ (d) $\{2, -2\}$

3. The S.S. of the equation : $x^2 + 25 = 0$ in $\mathbb{R}$ is ....
(a) $\{5\}$ (b) $\{5, -5\}$ (c) $\{-5\}$ (d) $\varnothing$

4. The equation whose roots are 3 and 5 is ....
(a) $5x^2 + 8x + 3 = 0$ (b) $2x^2 + 8x - 15 = 0$
(c) $x^2 - 8x + 15 = 0$ (d) $3x^2 + 8x + 5 = 0$

5. The S.S. of the equation : $x(x-3) = 5x$ in $\mathbb{R}$ is ....
(a) $\{3\}$ (b) $\{0, 3, 5\}$ (c) $\{3, 5\}$ (d) $\{0, 8\}$

6. The S.S. of the equation : $\frac{4}{x} = \frac{x}{9}$ in $\mathbb{R}$ is ……….

(a) $\{4 , 9\}$ (b) $\{6 , -6\}$ (c) $\{6\}$ (d) $\{36\}$

7. If the number 4 is a solution of the equation : $x^2 + x - 20 = 0$, then the other solution is ……….

(a) 20 (b) 5 (c) -5 (d) -4

Homework

1. The S.S. of the equation : $(x-4)^2 = 0$ in $\mathbb{R}$ is ……….

(a) $\{4\}$ (b) $\{0 , 4\}$ (c) $\{0 , -4\}$ (d) $\{-4\}$

2. The solution set of the equation : $x(x-3) = 0$ in $\mathbb{R}$ is ……….

(a) $\{3\}$ (b) $\{0 , 3\}$ (c) $\{0 , -3\}$ (d) $\{0\}$

3. If $3x^2 + cx - 6 = (3x-2)(x+3)$, then c = ……….

(a) 7 (b) 12 (c) 13 (d) 5

4. The expression : $x^2 + 6x + a$ is a perfect square when a = ……….

(a) 6 (b) 16 (c) 1 (d) 9

5. $x^3 + y^3 = (\cdots\cdots\cdots)(x^2 - xy + y^2)$

(a) $x^2 + y^2$ (b) $x^2 - y^2$ (c) $x + y$ (d) $x - y$

6. One of the factors of the expression : $x^2 - 3x - 18$ is ……….

(a) $x - 3$ (b) $x - 6$ (c) $x - 9$ (d) $x - 18$

Find in R the solution set of each of the following equations:

1. $x^2 - 7x - 30 = 0$

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2. $2x^2 + 7x = 0$

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3. $(x+2)^2 = 25$

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4. $(x-3)(x+5)=20$

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5. $x-\frac{2}{x}=\frac{7}{2}$

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6. $x(x-1)=6$

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7. $3x^3=12x$

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8. $x^3-4x=0$

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9. $x^4-13x^2+36=0$

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10. If : $x^2+\frac{1}{x^2}=34$, then find : $x+\frac{1}{x}$

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11. If : $x+\frac{1}{x}=2$, then find : $x^2+\frac{1}{x^2}$

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Homework

1. $x^2-5x-6=0$

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2. $x^2-6x=-9$

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3. $x-\frac{3}{x}=2$

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4. $x^2-5x=0$

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5. $4x^2=25$

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Lesson (9)

Applications on solving quadratic equations in one variable algebraically

Complete each of the following :

1. Twice the square of the number x is ………

2. If the age of Bassim now is x years , then his age 3 years ago was ……… years.

Choose the correct answer :

1. If the age of Ayman 5 years ago was x years , then the square of his age now = ………
(a) $x^2 + 5$ (b) $x^2 + 25$ (c) $(x + 5)^2$ (d) $(x - 5)^2$

2. If the age of Bassim now is x years , then his age 3 years ago was ……… years.
(a) $3x$ (b) $x + 3$ (c) $x - 3$ (d) x^3

3. If the age of Amgad now is x years , then his age after 7 years will be ……… years.
(a) $7x$ (b) $x - 7$ (c) $x + 7$ (d) x^7

4. If the age of Ayman 5 years ago was x years , then his age now is ……… years.
(a) $x - 5$ (b) $x + 5$ (c) $5x$ (d) $\frac{x}{5}$

5. If the age of Sally 2 years ago was x years , then her age after 3 years from now will be ……… years.
(a) $x + 2$ (b) $x + 3$ (c) $x + 5$ (d) $6x$

6. If the age of Magdy now is x years , then the square of his age after 2 years is ………
(a) $x^2 + 2$ (b) $x^2 + 4$ (c) $(x - 2)^2$ (d) $(x + 2)^2$

7. If the age of Samy now is x years , then twice his age 5 years ago is ……… years.
(a) $x - 5$ (b) $2x - 5$ (c) $x - 10$ (d) $2x - 10$

8. Three times the square of the number x is ………
(a) $(3x)^2$ (b) $x^2 + 3$ (c) $3x^2$ (d) $\frac{x^2}{3}$

Essay problems:

1. A positive integer whose square is more than five times the number by 36

Find the number.

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2. An integer , if we add twice its square to the number 7 the result will be 135

Find the number.

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3. Find the real number whose double exceeds its multiplicative inverse by one.

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4. Find two real numbers whose product is 45 and one of them is 4 more than the other.

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5. The sum of the squares of two successive odd numbers is 130

Find the two numbers.

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6. The sum of three successive integers is equal to the square of their middle integer.

Find these numbers.

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7. Hatem is 4 years older than Hanan now , and the sum of squares of their ages now is 26

Find their ages now.

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8. A right-angled triangle, the lengths of the two sides of the right angle are $4x$ cm. and $x + 1$ cm. If the area of the triangle = 84 cm^2 , calculate the length of its hypotenuse.

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Homework .

9. What is the real number which exceeds its multiplicative inverse by $\frac{5}{6}$?

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10. Find the rational number whose four times its square equals 81

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11. What is the real number if it is added to its square , the result will be 12 ?

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12. Find the dimensions of a rectangle whose length is 4 cm. more than its width and whose area is 21 cm^2

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Lesson (10)
Integer powers in R

Non-negative integer powers in $\mathbb{R}$

If $a \in \mathbb{R}$, $n \in \mathbb{Z}^+$, then $a^n = a \times a \times a \times a \times a \times \cdots \times a$ where a is repeated as a factor n times.
The symbol (a^n) is read as : a to the power n or the n^{th} power of the number a or the base a

Negative integer powers in $\mathbb{R}$

If a is a real number , $a \neq 0$ and n is a positive integer , then :

$$\boxed{a^{-n} = \frac{1}{a^n} \quad \text{and} \quad a^n = \frac{1}{a^{-n}}}$$

If $a \in \mathbb{R}^*$ (The set of non-zero real numbers) , then : $\boxed{a^0 = 1}$

$$(-a)^n = a^n \qquad \text{if n is an even number}$$

$$(-a)^n = -a^n \qquad \text{if n is an odd number}$$

Remarks

1 For every $a \in \mathbb{R}^*$, $n \in \mathbb{Z}^+$, then $\boxed{a^n \times a^{-n} = a^n \times \frac{1}{a^n} = 1 \text{ (the multiplicative neutral)}}$

i.e. a^n and a^{-n} are the multiplicative inverse of each other.

2 For every $a \in \mathbb{R}^*$, $b \in \mathbb{R}^*$ and $n \in \mathbb{Z}^+$, then $\boxed{\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n}$

For example : $\left(\frac{2}{3}\right)^{-2} = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$

Complete each of the following :

1.	$(a^2 b^{\cdots\cdots})^4 = a^8 b^{12}$
2.	If $(x-5)^{zero} = 1$, then : $x \in$ ....
3.	If $a = 7^x$ and $b = 7^{-x}$, then : $a \times b =$ ....
4.	If $x = \left(\sqrt{2}+3\right)^5$ and $y = \left(\sqrt{2}+3\right)^{-5}$, then : $x y =$ ....
5.	$\left(\frac{5}{6}\right)^{-4} = \left(-\frac{\cdots\cdots\cdots}{\cdots\cdots\cdots}\right)^2$

6.	If $\left(\frac{1}{2}\right)^{x} = 5$, then : $(8)^{-x} = \dots\dots\dots$
7.	If $2^{x} = 7$, $2^{y} = 5$, then : $2^{x+y} = \dots\dots\dots$
8.	If $5^{x} = 3$, $5^{-y} = 7$, then : $5^{x+y} = \dots\dots\dots$

Choose the correct answer :

1. $5^2 + 5^2 = \dots\dots\dots$

(a) 10^2 (b) 10^4 (c) 5^4 (d) 50

2. $3^5 \times 2^5 = \dots\dots\dots$

(a) 5^{10} (b) 6^{10} (c) 6^5 (d) 6^{25}

3. $(5\,a)^{zero} = \dots\dots\dots$, $a \neq 0$

(a) 5 (b) a (c) 5 a (d) 1

4. $3\,x^{zero} = \dots\dots\dots$, $x \neq 0$

(a) zero (b) 1 (c) 3 (d) $3\,x$

5. $3^{(2^3)} = \dots\dots\dots$

(a) 3^6 (b) 3^5 (c) 3^8 (d) 3^{32}

6. 🕮 $4^3 + 4^3 + 4^3 + 4^3 = \dots\dots\dots$

(a) 4^3 (b) 4^4 (c) 4^{12} (d) 4^{81}

7. The quarter of the number $4^{20} = \dots\dots\dots$

(a) 1^{20} (b) 4^{19} (c) 4^{16} (d) 4^5

8. 4 times the number $2^8 = \dots\dots\dots$

(a) 2^{32} (b) 8^8 (c) 2^{10} (d) 4^8

9. $\left(\sqrt{3}\right)^6 \times 3^4 = \dots\dots\dots$

(a) $\left(\sqrt{3}\right)^{24}$ (b) 3^{10} (c) 3^7 (d) $\left(\sqrt{3}\right)^{10}$

10. 🕮 The value of : $2^{20} + 2^{21} = \dots\dots\dots$

(a) 2×2^{40} (b) 2×2^{41} (c) 3×2^{20} (d) 3×2^{21}

11. 🕮 What of the following is closest to $11^2 + 9^2$?

(a) $22 + 18$ (b) $211 + 29$ (c) $120 + 20$ (d) $120 + 80$

12. 🕮 If $5^{x} = 4$, then $5^{x-1} =$ ....

(a) 1.25 (b) 0.8 (c) 0.125 (d) 0.08

13. 🕮 $0.002 \times 0.05 =$ ....

(a) 10^{-5} (b) 10^{-4} (c) 10^{4} (d) 10^{5}

14. $x^{m-1} \times$ .... $= 1$, $x \neq 0$

(a) x^{m-1} (b) x^{-m-1} (c) x^{m+1} (d) x^{-m+1}

15. $5 \times 5 \times 5 \times 2 \times 2 \times 2 \times 2 \times 2 = 4 \times$ ....

(a) 5^3 (b) 2^3 (c) 10^3 (d) $5^3 + 2^3$

Homework

1. $(5^2)^3 =$ ....

(a) 5^6 (b) 5^5 (c) 5^{32} (d) 5

2. $2^5 + 2^5 + 2^5 + 2^5 =$ ....

(a) 2^4 (b) 2^6 (c) 2^7 (d) 2^{20}

3. 🕮 Sixth the number $2^{12} \times 3^{12}$ is ....

(a) 6^2 (b) 6^4 (c) 6^{11} (d) 6^{23}

4. Fifth the number $\left(\sqrt[3]{5}\right)^6$ is ....

(a) 5 (b) 5^5 (c) 5^6 (d) 5^{12}

5. 🕮 The value of : $2^5 + \left(\sqrt{2}\right)^{10} =$ ....

(a) 2^6 (b) 2^{10} (c) $\left(\sqrt{2}\right)^{15}$ (d) $\left(\sqrt{2}\right)^{20}$

6. If $6^{x} = 11$, then $6^{x+1} =$ ....

(a) 12 (b) 22 (c) 66 (d) 72

7. 🕮 If $x = \frac{\sqrt{9}}{\sqrt{3}}$, then $x^{-1} =$ ....

(a) $\frac{\sqrt{3}}{3}$ (b) $\frac{\sqrt{3}}{\sqrt{2}}$ (c) $\sqrt{3}$ (d) 2

8. 📖 $\left(\sqrt{3}+\sqrt{2}\right)^{9}\left(\sqrt{3}-\sqrt{2}\right)^{9} = \dots\dots\dots$

(a) 1 (b) $\sqrt{5}$ (c) $\sqrt{6}$ (d) 5

9. 📖 The numerical value of the expression : $\frac{2^{2n+1} \times 5^{2n+1}}{10^{2n}}$ is ………

(a) $\frac{1}{10}$ (b) 7 (c) 10 (d) 100

10. $2^{2011} = 2^{2010} + \dots\dots\dots$

(a) 2 (b) 2010 (c) 2^{2010} (d) 2^{2011}

Find the value of each of the following in the simplest form:

1. 3^{-2}
2. $\left(\sqrt{5}\right)^{4}$
3. $\left(-\sqrt{3}\right)^{-2}$
4. $(0.01)^{-2}$
5. $\left(x^{2}\right)^{-3} \times \left(x^{-3}\right)^{-2}$
6. $\frac{\left(x^{2}\right)^{-3} \times \left(x^{-1}\right)^{2}}{x^{-3} \times x^{-4}}$
7. $\left(-\sqrt{5}\right)^{9} \div \left(-\sqrt{5}\right)^{5}$
8. $\left(\left(\sqrt{2}\right)^{3} \times \left(-\sqrt{2}\right)^{2}\right)^{2}$
9. $\left(\sqrt{3}\right)^{-4} \times \left(-\sqrt{2}\right)^{4}$
10. $\left(\left(-5\right)^{3}\right)^{2} \times \left(-\sqrt{5}\right)^{-4}$

11.	$\dfrac{(\sqrt{7})^{-4} \times (\sqrt{7})^{-3}}{(\sqrt{7})^{-9}}$
12.	$\dfrac{(\sqrt{3})^{5} \times (\sqrt{3})^{4}}{(\sqrt{3})^{3} \times 27}$
13.	$\dfrac{(10)^{2} \times (10)^{-7}}{(0.1)^{2} \times 0.001}$
14.	$\left(\dfrac{3\sqrt{2}}{2\sqrt{3}}\right)^{4}$
15.	$\dfrac{9^{x} \times 3^{x+2}}{(27)^{x}}$
16.	$\dfrac{(36)^{n} \times 5^{2n}}{(30)^{2n}}$
17.	$\dfrac{8^{n-1} \times 32^{-n}}{32 \times 4^{-n}}$
18.	$\dfrac{6^{n} \times 4^{n+\frac{1}{2}}}{(24)^{n}}$
19.	If $\dfrac{8^{x} \times 9^{x}}{18^{x}} = 64$, find the value of 4^{-x} ....
20.	🕮 If $a = \sqrt{3}$ and $b = \sqrt{2}$, find the value of : **1** $a^4 - b^4$ **2** $\dfrac{a^4}{b^4}$

21. If $x = 2\sqrt{2}$ and $y = 3$, find the value of : $(x^2 - y^2)^3$

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Homework

1.	$\left(\frac{1}{4}\right)^{-1}$
2.	$\left(\frac{\sqrt{3}}{3}\right)^{-5}$
3.	$x^3 \times x^{-2} \times x^{-1}$
4.	$(\sqrt{2})^2 \times (\sqrt{2})^4$
5.	$\left(\frac{-1}{\sqrt{2}}\right)^6$
6.	$\frac{(\sqrt{3})^7 \times (\sqrt{3})^8}{(\sqrt{3})^6}$
7.	$\frac{(\sqrt{5})^{10} \times (-\sqrt{5})^5}{(\sqrt{5})^{11}}$
8.	$\frac{2^x \times 4^{x+1}}{8^x}$
9.	$\frac{4^n \times 6^{2n}}{2^{4n} \times 3^{2n}}$

Lesson (11)

Solving exponential equations in R

If a is a real number , m and n are two integers

and $\boxed{a^m = a^n}$, then $\boxed{m = n}$ where : $a \neq 0$, $a \neq \pm 1$

For example :

If $3^n = 9$, then : $3^n = 3^2$

, $\because$ the base = the base $\therefore$ the power = the power $\therefore$ $\boxed{n = 2}$

If a and b are two real numbers , m is an integer and $\boxed{a^m = b^m}$, then :

- $\boxed{a = b}$ if m is an odd number. ***For example :*** If $n^5 = 3^5$, then : $n = 3$
- $\boxed{a = \pm b}$ if m is an even number. ***For example :*** If $n^2 = 3^2$, then : $n = \pm 3$
- $\boxed{m = \text{zero}}$ if $a \neq \pm b$

For example : If $7^{n-2} = 5^{n-2}$, then : $n - 2 = 0$ $\therefore n = 2$

Complete each of the following :

1.	If $5^{x(x-1)} = 1$, then the value of x = ...
2.	If $3^n \times 3^5 = 1$, then n = ...
3.	If $3^x + 3^x + 3^x = 1$, then x = ...
4.	If $\{3 , a^{x-2}\} = \{1 , 3\}$, then the value of x = ...
5.	If $(2^x , 125) = (16 , y^3)$, then x = ... and y = ...

Homework

1.	If $2^y \times 5^y = 100$, then y = ...
2.	If $\left(\frac{3}{5}\right)^{x-7} = 1$, then x = ...

Choose the correct answer :

1. If $3^{x+1} = 5^{x+1}$, then $x =$ ....

(a) 4 (b) 3 (c) – 1 (d) 1

2. If $3^{2+x} = 5^{x+2}$, then $7^{x+2} =$ ....

(a) 7 (b) – 7 (c) – 14 (d) 1

3. If $\left(\frac{2}{3}\right)^9 = \left(\frac{3}{2}\right)^x$, then $x =$ ....

(a) – 9 (b) 9 (c) 32 (d) 23

4. If $5^{|x-3|} = 25$, then $x =$ ....

(a) 5 (b) 2 (c) 1 (d) 5 or 1

5. 🕮 If $2^{x-1} \times 3^{1-x} = \frac{9}{4}$, then $x =$ ....

(a) – 3 (b) – 1 (c) 1 (d) 3

Homework

1. If $2^x = \frac{1}{8}$, then $x^2 =$ ....

(a) $\frac{1}{4}$ (b) 9 (c) – 9 (d) $-\frac{1}{9}$

2. If $2^{x-2} = 2^{1-2x}$, then $x =$ ....

(a) 2 (b) $\frac{1}{2}$ (c) 1 (d) zero

3. If $3^x = 9$, then $2^x - 1 =$ ....

(a) 7 (b) 3 (c) 8 (d) 5

4. If $3^x = 7$, $7^y = 9$, then $x\,y =$ ....

(a) 5 (b) 2 (c) 7 (d) 9

Essay problems:

Find the value of n in each of the following when $n \in \mathbb{Z}$:

1. $3^{n-2} = 81$

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2. $\left(\sqrt{3}\right)^{n-1} = 9$

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3. $\left(\frac{3}{5}\right)^{n+2} = \frac{125}{27}$

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4. $\left(\frac{2}{3}\right)^{n-4} = 2\frac{1}{4}$

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5. $\left(\frac{2}{3}\right)^{n+5} = \left(3\frac{3}{8}\right)^{-2}$

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6. $\dfrac{2^{n} \times 9^{n+1}}{(18)^{n}} = 3^{n}$

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7. $\dfrac{(12)^{n-1}}{2^{n-1} \times 3^{n-1}} = 1$

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8. $\dfrac{(14)^{2n} \times 4^{n+1}}{4 \times 7^{n} \times 16^{n}} = 49$

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Homework

1. $3^{n-2} = \dfrac{1}{9}$

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2. $\left(\dfrac{2}{5}\right)^{2n-1} = \dfrac{8}{125}$

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3. $\frac{3^n \times 8^n}{(12)^{n+1}} = \frac{1}{3}$

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Find the S.S. of each of the following equations in $\mathbb{R}$:

1. $3^{x-3} = (\sqrt{3})^{x+5}$

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2. $25 \times 3^{x-1} = 9 \times 5^{x-1}$

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3. $5^{x^2 - 5x} = 0.0016$

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4. $\frac{1}{(x+9)^4} = 0.0001$

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5. $9^{x^2-1} = \frac{1}{(27)^x}$

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6. If $\left(\sqrt{\frac{3}{2}}\right)^x = \frac{4}{9}$, calculate the value of : $\left(\frac{3}{2}\right)^{x+1}$

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Homework

1. $(32)^{x-3} = 8^{2x+1}$

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2. If $\frac{49^n \times 25^{2n} \times 3^{4n}}{7^{-n} \times 15^{4n}} = 343$, then calculate the value of : 6^{2n}

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3. If $3^x = 27$, $4^{x+y} = 1$, calculate the value of each of : x and y

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Lesson (12)
Operations on integer powers

1	2	3	4
Do the operations inside the parenthesis (the interior, then the exterior).	Calculate the powers of the numbers (indices).	Do the multiplication and the division in order from left to right.	Do the addition and the subtraction in order from left to right.

Complete each of the following :

1. The simplest form of the expression : $2^{-3} \times 2^{-2} \div 4^{-3} =$ ....

2. The simplest form of the expression : $4^3 \times 3^{-2} \times \left(\sqrt[3]{-8}\right)^{-5} =$ ....

Homework

1. The simplest form of the expression : $2^{-3} \times 3^{-2} \div 6^{-4} =$ ....

2. The simplest form of the expression : $\left(3^{-2}\right)^3 \div 9^{-3} \times (-2)^{-1} =$ ....

Choose the correct answer :

1. 🕮 The expression : $\dfrac{3^x \times 3^x \times 3^x}{3^x + 3^x + 3^x}$ equals ....

(a) 3^{2x-1} (b) 3^{1-2x} (c) 3^{x^3-3x} (d) 3^{3x-x^3}

2. 🕮 $(5^{x+2} - 5^{x+1}) \div 5^x =$ ....

(a) 5 (b) 10 (c) 15 (d) 20

Homework

1. 🕮 The value of the expression : $3^5 + \left(\sqrt{3}\right)^{10} - 2\,(3)^5 =$ ....

(a) zero (b) 3^5 (c) $\left(\sqrt{3}\right)^5$ (d) $2\,(3)^5$

2. The simplest form of the expression : $\sqrt{4 \times \sqrt{16} \div \sqrt[3]{8} - 2^2} =$ ....

(a) 2 (b) 4 (c) 8 (d) 16

3. If $x = \sqrt{3}$, $y = \sqrt{5}$, then : $\dfrac{x^8 - y^8}{x^4 + y^4} =$ ....

(a) 4 (b) -4 (c) 16 (d) -16

Find the result in the simplest form:

1. $\left(\sqrt{5}\right)^{5} \div 5\sqrt{5} + 2\sqrt{3} \times \sqrt{3}$

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2. $\left(\sqrt{3}\right)^{-3} \times 3\sqrt{3} + \left(\sqrt{3}\right)^{-4} \div \left(\sqrt{3}\right)^{-10}$

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3. $\dfrac{\left(\sqrt{3}\right)^{7} \times \left(\sqrt{3}\right)^{-5} - \left(\sqrt{3}\right)^{2}}{\left(\sqrt{3}\right)^{7} \times \left(\sqrt{3}\right)^{-5} + \left(\sqrt{3}\right)^{2}}$

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4. **If $a = \sqrt{2}$, $b = \sqrt{3}$, find the numerical value of :**

1 $\dfrac{b^4 - a^4}{b^2 + a^2}$ « 1 »

2 $\dfrac{a^3 + b^3}{a + b}$ « $5 - \sqrt{6}$ »

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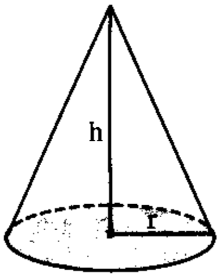
5. If the volume of the right circular cone is given by the relation : $v = \frac{1}{3} \pi r^2 h$

Find the height of the cone h if the volume is : 7.7×10^2 cm^3 and its diameter length is 14 cm. $\left(\pi = \frac{22}{7}\right)$



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Homework

1. $\left(2\sqrt{3}\right)^3 \times \sqrt{3} - \left(\sqrt{2}\right)^7 \div 4\sqrt{2}$

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2. $\left(2\sqrt{5}\right)^4 - \left(\sqrt{5}\right)^{-3} \times \left(5\sqrt{5}\right)^2 \div 5\sqrt{5}$

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Lesson (13)
The probability

The probability of occurrence of a certain event $= \dfrac{\text{the number of times of repeating this outcome}}{\text{the number of all possible outcomes}}$

The expected number for occurrence of a certain event
= the probability of its occurrence × the total number of given individuals

The random experiment is an experiment , where all its possible outcomes are known before doing it but we can't determine the actual outcome.

The sample space is the set of all possible outcomes of a random experiment and it is denoted by S.

The number of its elements is denoted by n (S)

The event

It is a subset of the sample space.

The probability of occurrence of an event $A \subset S$ is denoted by P (A)

It is found by using the relation :

$$P(A) = \frac{\text{the number of elements of A}}{\text{the number of elements of the sample space}} = \frac{n(A)}{n(S)}$$

Remarks

1 **The impossible event :** is the event which cannot occur.

i.e. The probability of the impossible event equals zero.

2 **The certain event :** is the event whose outcomes are all possible.

i.e. The probability of the certain event = 1

3 The probability of any event is not less than zero and it is not more than 1

i.e. For any event A , $0 \le P(A) \le 1$ *i.e.* $P(A) \in [0 , 1]$

Complete each of the following :

1. For every event A , we find that P (A) ∈ ....

2. 10 cards are numbered from 1 to 10 A card is drawn randomly , then the probability that the card carries a prime number = ...

3. A box contains 5 white balls , 7 red balls and 3 blue balls. If a ball is drawn from the box randomly , then the probability that the drawn ball is blue = ...

4. In the experiment of throwing a fair die and observing the number on the upper face , then the probability of getting a number less than 1 equals ....

5. A box contains 48 oranges , 4 of them are bad. If we draw an orange at random , then the probability that the drawn orange is bad = ...
and the probability that it is not bad = ...

6. A city has 200000 people. The probability that a person gets infected by a disease in this city is 0.003 The expected number of infection is .... people.

7. A factory produces 400 lamps daily , if the probability that the lamp is defective = 0.02 , then the expected number of good lamps produced daily is ....

Homework

1. The probability of the impossible event = ...
and the probability of the certain event = ...

2. If a fair coin is tossed once , then the probability of appearance of a head = ...

3. A bag contains 12 balls , 4 of them are red , 6 are green and the rest are blue. If one ball is chosen randomly , then the probability of getting a blue ball = ...

4. In the experiment of throwing a fair die and observing the number on the upper face , then the probability of getting a number greater than 4 is ....

5. If the probability of the occurrence of an event is $\frac{5}{8}$, then the probability of the non-occurrence of this event is ....

6. A room has 3 doors numbered from 1 to 3 One student goes out from one door. The probability that he goes out from the second door is ....

Choose the correct answer :

1. Which of the following may be the probability of an event ?
(a) 1.2 (b) – 0.4 (c) 315% (d) 75%

2. A basket has cards labelled by the numbers from 1 to 20. If a card is selected randomly, what is the probability that the number labelled on the card is divisible by 6 ?

(a) $\frac{3}{20}$ (b) $\frac{4}{20}$ (c) $\frac{5}{20}$ (d) $\frac{6}{20}$

3. In a competition between two players if the probability that the first player win is 0.25 , then the probability of the second player win is ……… (The competition continues till one of two players win)

(a) zero (b) 0.25 (c) 0.75 (d) 1

4. Ahmed is a pupil in 2^{nd} preparatory. In his class, there are 36 pupils.16 of them are girls. If a pupil is selected randomly, what is the probability that the pupil is a boy ?

(a) $\frac{4}{9}$ (b) $\frac{1}{2}$ (c) $\frac{5}{9}$ (d) $\frac{1}{36}$

5. A student is asked to draw a triangle choosing from the three types (acute – angled triangle or right – angled triangle or obtuse – angled triangle) freely , then the probability that the student draw a right – angled triangle is ………

(a) 3 (b) $\frac{1}{3}$ (c) $\frac{2}{3}$ (d) $\frac{1}{6}$

6. A bag contains a number of similar balls, half of them are red, $\frac{1}{3}$ of them are black and the rest are white. One ball is chosen. The probability that the chosen ball is white equals ………

(a) $\frac{1}{2}$ (b) $\frac{1}{6}$ (c) $\frac{1}{3}$ (d) zero

7. If the probability that worker go to his work on foot is twice the probability of using any other mean of transport , then the probability that the worker use a mean of transport = ………

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{2}{3}$ (d) 2

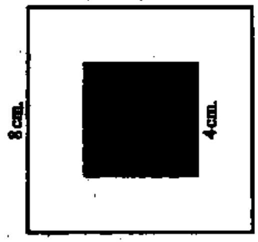
8. A box contains balls coloured with red, green, blue and yellow. If the box contains 20 yellow balls and the probability of selecting a yellow ball randomly is $\frac{1}{4}$, what is the number of balls in the box ?

(a) 5 (b) 25 (c) 60 (d) 80

9. In a mixed school , there are 1500 pupils. A random sample formed from 200 pupils is selected. It is found that the number of girls equals 90. What is the expected number of girls in the school ?

(a) 600 girls (b) 625 girls (c) 650 girls (d) 675 girls

10. In the opposite board two squares are drawn , if a person points at it as a target , then the probability of hitting the shaded region is ....



(a) $\frac{1}{2}$ (b) $\frac{1}{3}$

(c) $\frac{1}{4}$ (d) $\frac{1}{8}$

Homework .

1. If the probability that a pupil succeeds is 70% , then the probability of his failure is ....

(a) 0.7 (b) 0.07 (c) 0.3 (d) 0.03

2. In an experiment of throwing a fair die , then the probability of appearing a number not equal to 2 in the upper face is ....

(a) $\frac{1}{6}$ (b) $\frac{2}{3}$ (c) $\frac{1}{3}$ (d) $\frac{5}{6}$

3. If a coin is thrown 400 times , then the most expected number of appearing tail is ....

(a) 204 (b) 199 (c) 240 (d) 195

4. There are 25 boys and 20 girls in a classroom. One pupil is chosen randomly. The probability that the chosen pupil is a girl equals ....

(a) $\frac{1}{20}$ (b) $\frac{4}{9}$ (c) $\frac{1}{25}$ (d) $\frac{5}{9}$

5. A bag contains 3 white balls, 2 black balls and one red ball. A ball is selected randomly from the bag. Then the probability that the selected ball is not black equals ....

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{2}{3}$ (d) $\frac{1}{6}$

6. The number of pupils in a class of 2^{nd} year preparatory is 36 pupils, the probability of selecting a pupil whose age is less than or equal to 13 years is $\frac{1}{6}$

What is the number of pupils in the class whose ages are more than 13 years ?

(a) 23 (b) 24 (c) 30 (d) 32

7. In a mixed school, the ratio between the number of boys to the number of girls is 7 : 9
A pupil is selected randomly from this school.
The probability that the selected pupil is a boy equals ………

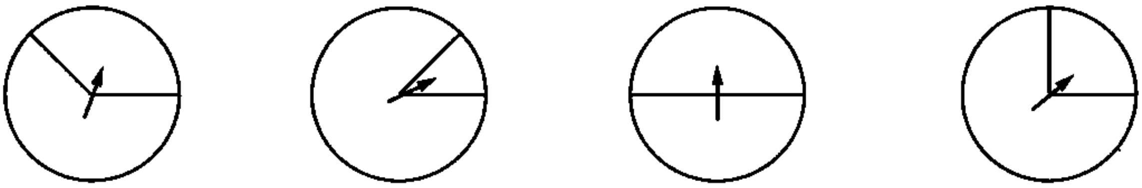
(a) zero (b) $\frac{7}{16}$ (c) $\frac{9}{16}$ (d) 7

8. The following table shows the numbers of 160 pupils in a school who like to practise a certain game. If a pupil is selected randomly from this sample, what is the probability that he is practising handball ?

Game	Swimming	Handball	Athletics	Football	Gymnastics	Boxing
Number	20	40	30	50	10	10

(a) $\frac{1}{16}$ (b) 25% (c) $\frac{1}{4}$% (d) $\frac{5}{16}$

9. A spinner game is divided into two parts x and y, then the point is rolled 400 rounds, it stopped 98 times in the region x, then which of the following figures the pointer points to the region x ?



(a) (b) (c) (d)

Essay problems:

1. Selecting randomly a card out of 40 similar cards in a box numbered from 1 to 40
Find the probability of getting a card that carries :

1. an even number
2. a number divisible by 3
3. a number is not divisible by 10
4. an even number is divisible by 3
5. a prime number less than 20

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2. **Drawing randomly a coloured marble out of a box containing 12 red marbles, 18 white marbles and 20 blue marbles. Find the probability of selecting :**

1. a white marble.
2. a red marble.
3. a yellow marble.
4. a non-red marble.
5. a red or blue marble.

....

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3. A box contains 80 similar balls. Some of them are red and the remained are blue.
If the probability of drawing a red ball is $\frac{1}{4}$, find the number of blue balls.

....

4. A garment factory in the Tenth of Ramadan City produces 6000 units daily. As a sample of 1000 units was examined, 20 defective units were found. Calculate the number of defective units.

....

5. In a fruit packing plant, 30% of fruits is not suitable for exporting because the size is too small. How many tons can be exported in 10 days if 20 tons of fruits are delivered back daily to the factory ?

....

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6. A calculator manufacturing company examined randomly electronic circuits in a sample of 200 units. The defective production was 6%

1. How many units are out of order in this sample ?
2. If the total production in one month was 1500 units, how many units are functional units of marketing ?

....

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....

7. **A survey has been conducted on 100 students about their favourite games which they practise. The result was as follows :**

Favourite game	Football	Handball	Athletics	Tennis	Hockey
Number of students	44	27	12	4	13

1 Find the probability if a student prefers :

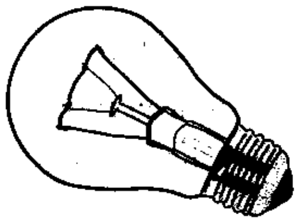
(a) Practising football. (b) Practising handball. (c) Practising athletics.

(d) Practising tennis. (e) Practising hockey.

2 If the number of students is 600, how many students are predicted to practise hockey ?

....

8. **In producing 300 electric lamps, 18 units were found defective.**



1 What is the probability of a unit to be a defective unit ?

2 What is the probability of a functional unit ?

3 Is it possible for a unit to be a functional unit and out of order unit at the same time ?

4 Find the sum of the probability of a defective unit and the probability of a functional unit. What do you observe ?

5 If a daily production of this factory was 1600 electric lamps, find the number of the functional units in that day.

....

Homework

1. **A numbered card is selected randomly from a set of similar cards numbered from 1 to 24 Find the probability of getting a card that carries :**

1 a multiple of 4

2 a multiple of 6

3 a multiple of 4 and 6 together

4 a multiple of 4 or 6

5 a number divisible by 25

6 a positive integer less than 25

....

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2. A class has 50 students , the number of girls is less than the number of boys by 10
If a student is chosen randomly , find the probability that the student is a boy.

....

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3. 🕮 Drawing randomly a coloured marble out of a bag containing 32 similar marbles coloured red , white , green and yellow , the probability of getting a red marble is $\frac{3}{8}$
Estimate how many red marbles are in the bag.

....

4. 🕮 **The following table shows the evaluation of 50 students in one month :**

A student is randomly selected. What is the probability of getting a score of :

1 Excellent

2 Good

3 Failed

4 Less than good

Estimate	Excellent	Very good	Good	Pass	Fail
Number	6	9	11	16	8

GEOMETRY

SECOND TERM

PREPARATORY TWO

Lesson (14)
Equality of the areas of two parallelograms

Theorem 1

Surfaces of two parallelograms with common base and between two parallel straight lines , one is carrying this base , are equal in area.

Corollary 1

The parallelogram and the rectangle with common base and between two parallel straight lines are equal in area.

Corollary 2

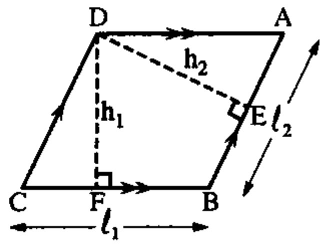
The area of the parallelogram = the length of the base × its corresponding height.

Remark

In the opposite figure :

If ABCD is a parallelogram , DF is the corresponding height of the base $\overline{BC}$ and DE is the corresponding height of the base $\overline{AB}$, then : The area of the parallelogram ABCD = BC × DF = AB × DE

i.e. $\boxed{\ell_1 \times h_1 = \ell_2 \times h_2}$



Corollary 3

The parallelograms with bases equal in length and lying on a straight line , while the opposite sides to these bases are on another straight line , are equal in area.

Corollary 4

Area of a triangle is equal to half of area of a parallelogram if they have a common base lying on one of two parallel straight lines including them.

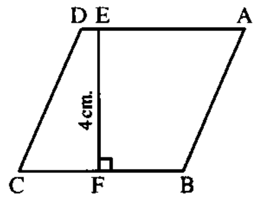
Corollary 5

Area of the triangle = $\frac{1}{2}$ of the length of the base × its corresponding height

Complete each of the following :

1. If the area of ▱ ABCD = 400 cm^2 ,

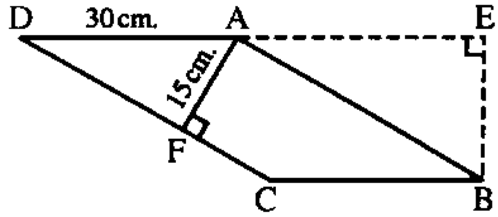
then BC = ... cm.



2. If the area of ▱ ABCD = 600 cm^2 ,

then CD = ... cm. ,

BE = ... cm.



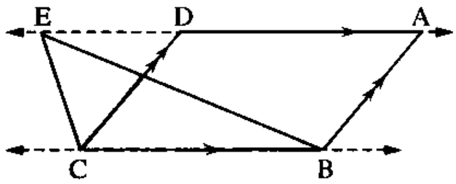
3. **In the opposite figure :**

ABCD is a parallelogram and E $\in \overleftrightarrow{AD}$

Complete the following :

1 The area of Δ EBC = ... the area of ▱ ABCD

2 If the area of Δ EBC = 20 cm^2 , then the area of ▱ ABCD = ... cm^2

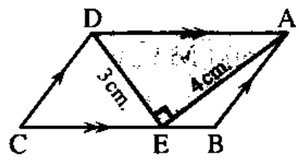


4. **In the opposite figure :**

ABCD is a parallelogram , AE = 4 cm. , ED = 3 cm.

, m (∠ AED) = 90° and E $\in \overline{BC}$ **Complete :**

1 The area of Δ AED = ... cm^2 **2** The area of ▱ ABCD = ... cm^2



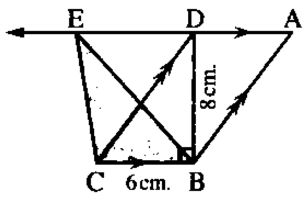
5. **In the opposite figure :**

ABCD is a parallelogram in which , BC = 6 cm. , $\overline{DB} \perp \overline{BC}$,

such that , DB = 8 cm. and E $\in \overrightarrow{AD}$

Complete :

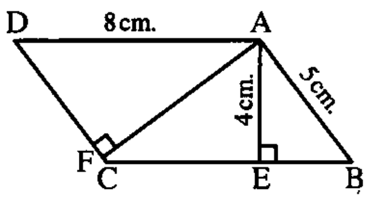
1 The area of ▱ ABCD = ... cm^2 **2** The area of Δ EBC = ... cm^2



Homework

1. If ABCD is a parallelogram ,

then AF = ... cm.



2. Surfaces of two parallelograms with common base and between two parallel straight lines , one is carrying this base , are ....

3. The parallelogram and ·········· with common base and between two parallel straight lines are equal in area.

4. The area of the parallelogram = ·········· × ··········

5. The areas of the parallelograms with bases equal in length and lying on a straight line , while the opposite sides to these bases are on another straight line , are ··········

Choose the correct answer :

1. If the base length of a parallelogram is 7 cm. and the corresponding height is 4 cm. , then its area = ··········

   (a) 11 cm^2 (b) 14 cm^2 (c) 22 cm^2 (d) 28 cm^2

2. If the area of a parallelogram is 35 cm^2 and its height is 5 cm. , then the length of the corresponding base is ··········

   (a) 5 cm. (b) 7 cm. (c) 9 cm. (d) 30 cm.

3. If ABCD is a parallelogram in which , AB = 5 cm. , BC = 10 cm. and its smaller height is 4 cm. , then its greater height = ··········

   (a) 2 cm. (b) 4 cm. (c) 8 cm. (d) 10 cm.

4. A parallelogram whose area = 50 cm^2 and the length of its base equals twice the corresponding height , then this height = ··········

   (a) 50 cm. (b) 25 cm. (c) 10 cm. (d) 5 cm.

5. The ratio between the area of the parallelogram and the area of the triangle whose base is common and are included between two parallel straight lines = ··········

   (a) 1 : 2 (b) 1 : 3 (c) 2 : 1 (d) 2 : 3

6. If the area of the triangle is 42 cm^2 and its height = 7 cm. , then the length of the corresponding base = ··········

   (a) 15 cm. (b) 12 cm. (c) 8 cm. (d) 4 cm.

7. The area of a right-angled triangle in which the lengths of the sides of the right angle are 6 cm. and 9 cm. equals ....

(a) 54 cm^2 (b) 60 cm^2 (c) 27 cm^2 (d) 15 cm^2

8. The area of the rectangle whose dimensions are 6 cm. and 4 cm. .... the area of the triangle whose base length is 12 cm. and the corresponding height is 4 cm.

(a) $<$ (b) $>$ (c) $=$ (d) $\neq$

Homework

1. If the area of a parallelogram is 50 cm^2 and its base length = 10 cm. , then the corresponding height of this base = ...

(a) 500 cm. (b) 5 cm. (c) 250 cm. (d) 100 cm.

2. If the lengths of two adjacent sides of a parallelogram are 8 cm. and 10 cm. and its greater height is 5 cm. , then its area = ...

(a) 80 cm^2 (b) 50 cm^2 (c) 40 cm^2 (d) 18 cm^2

3. The area of the triangle is .... the area of the parallelogram which has a common base with it and its vertex lies on the straight line parallel to this base.

(a) equal to (b) half (c) twice (d) quarter

4. The area of the triangle = ... the base length × the corresponding height.

(a) 2 (b) $\frac{1}{2}$ (c) $\frac{1}{4}$ (d) $\frac{1}{3}$

5. If the base length of a triangle is 4 cm. and the corresponding height = 3 cm. , then its area = ...

(a) 6 cm^2 (b) 12 cm^2 (c) 24 cm. (d) 34 cm^2

6. The triangle whose base length is 12 cm. and its area is 48 cm^2 , the corresponding height = ...

(a) 3 cm. (b) 4 cm. (c) 6 cm. (d) 8 cm.

7. If ABCD is a parallelogram with area 100 cm^2 and $E \in \overline{AD}$, then the area of Δ EBC = ...

(a) 25 cm^2 (b) 50 cm^2 (c) 100 cm^2 (d) 200 cm^2

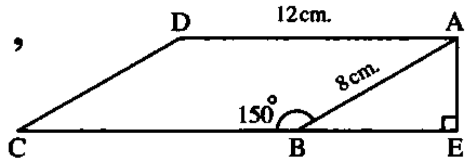
Essay problems:

1. **In the opposite figure :**

ABCD is a parallelogram in which $m(\angle ABC) = 150^\circ$,

$AD = 12$ cm.

, $AB = 8$ cm. , $E \in \overrightarrow{CB}$ and $\overline{AE} \perp \overrightarrow{CB}$



Find : The area of ▱ ABCD

« 48 cm^2 »

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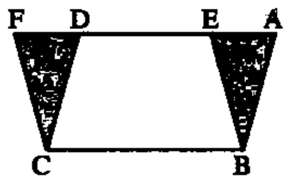
2. **In the opposite figure :**

ABCD and EBCF are two parallelograms ,

$E \in \overleftrightarrow{AD}$ and $F \in \overleftrightarrow{AD}$



Prove that : The area of Δ ABE = the area of Δ DCF

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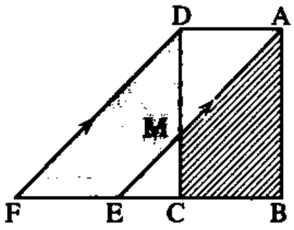
3. **In the opposite figure :**

ABCD is a rectangle , $\overline{AE} \parallel \overline{DF}$

Prove that :

The area of the figure ABCM = the area of the figure DMEF



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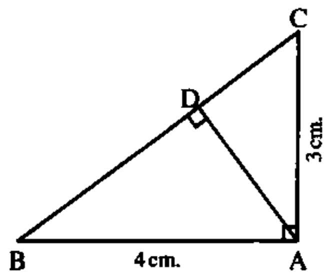
4. **In the opposite figure :**

ABC is a right-angled triangle at A ,

$\overline{AD} \perp \overline{BC}$, AB = 4 cm. and AC = 3 cm.

Find : 1 The area of Δ ABC 2 The length of $\overline{AD}$



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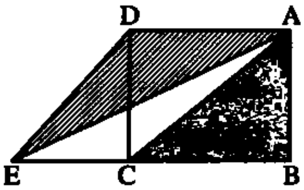
📖 **In the opposite figure :**



ABCD is a rectangle and $E \in \overrightarrow{BC}$

Prove that : The area of Δ DAE = the area of Δ ABC

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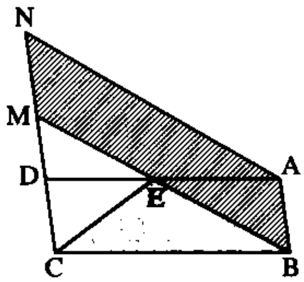

ABCD and ABMN are two parallelograms

and $M \in \overrightarrow{CD}$

Prove that :

The area of Δ EBC = $\frac{1}{2}$ the area of ▱ ABMN

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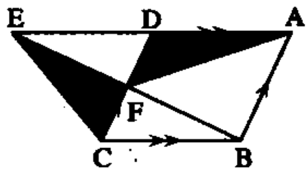
7. In the opposite figure :



ABCD is a parallelogram , $E \in \overrightarrow{AD}$ and $\overrightarrow{BE} \cap \overrightarrow{CD} = \{F\}$

Prove that : The area of Δ AFD = the area of Δ EFC

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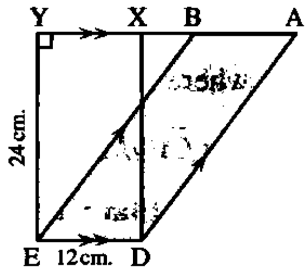
Homework

1. In the opposite figure :



$\overleftrightarrow{AB} \parallel \overleftrightarrow{DE}$, X and Y $\in \overleftrightarrow{AB}$

, XDEY is a rectangle and $\overline{AD} \parallel \overline{BE}$

1 Find the area of the figure ABED

2 If : AD = 30 cm. , find the length of the perpendicular from B to $\overline{AD}$

« 288 cm^2. , 9.6 cm. »

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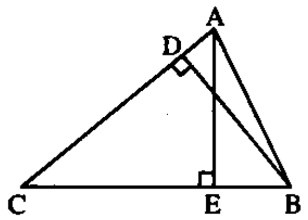
In the opposite figure :



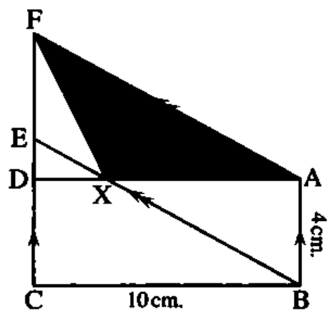
ABC is a triangle in which BC = 6.5 cm.

, AC = 6 cm. , $\overline{AE} \perp \overline{BC}$, $\overline{BD} \perp \overline{AC}$ and BD = 5 cm.

Find : **1** The area of Δ ABC **2** The length of $\overline{AE}$

3.

In the opposite figure :



ABCD is a rectangle , ABEF is a parallelogram

, D ∈ $\overline{CF}$, X ∈ $\overline{BE}$, E ∈ $\overline{CF}$

, AB = 4 cm. and BC = 10 cm.

Find by proof :

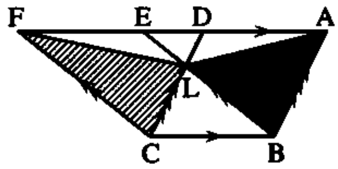
1 The area of ▱ ABEF

2 The area of Δ XAF

« 40 cm^2 , 20 cm^2 »

4. **In the opposite figure :**

ABCD and EBCF are two parallelograms , $\overline{BE} \cap \overline{CD} = \{L\}$, $D \in \overline{AF}$ and $E \in \overline{AF}$



Prove that :

1 The area of Δ ABL = the area of Δ FCL

2 The area of the figure ABCL = the area of the figure FCBL

Lesson (15)
Equality of the areas of two triangles

Theorem 2

Two triangles which have the same base and the vertices opposite to this base on a straight line parallel to the base have the same area.

Corollary 1

Triangles of bases equal in length and lying between two parallel straight lines are equal in area.

Corollary 2

The median of a triangle divides its surface into two triangular surfaces equal in area.

Corollary 3

Triangles with congruent bases on one straight line and have a common vertex are equal in areas.

Theorem 3

If two triangles are equal in area and drawn on the same base and on one side of it , then their vertices lie on a straight line parallel to this base.

Complete each of the following :

1. If ABC is a triangle , D is the midpoint of $\overline{BC}$, then :
The area of Δ ABD = the area of Δ ....

2. If $\overline{XL}$ is a median in Δ XYZ , then the area of Δ XYZ = ... the area of Δ XYL

3. The triangle XYZ in which L $\in \overline{YZ}$ such that YL = $\frac{1}{2}$ LZ , then:
The area of Δ XYL = ... the area of Δ XYZ

Homework

1. The two triangles drawn on a common base and their vertices located on a straight line parallel to the base are ....

2. Triangles with congruent bases and drawn between two parallel lines are ....

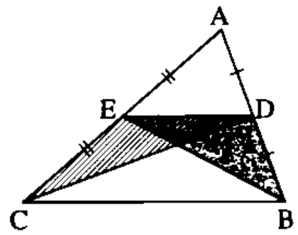
3. The median in the triangle divides its area into ....

Essay problems:

1. **In the opposite figure :**

D is the midpoint of $\overline{AB}$ and E is the midpoint of $\overline{AC}$

Prove that : The area of Δ BDE equals the area of Δ CDE



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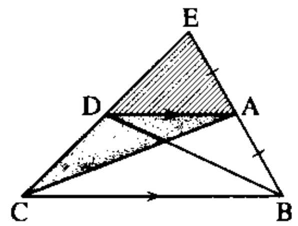
2. **In the opposite figure :**

ABCD is a quadrilateral in which $\overline{AD} \parallel \overline{BC}$ and $\overrightarrow{BA} \cap \overrightarrow{CD} = \{E\}$

such that BA = AE

Prove that : The area of Δ ADC = the area of Δ ADE



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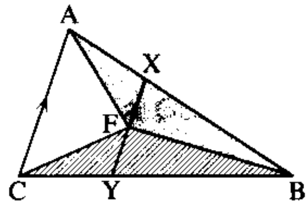
3. 📖 **In the opposite figure :**

$\overline{AC}$ // $\overline{XY}$ and F is the midpoint of $\overline{XY}$

Prove that : The area of Δ ABF = the area of Δ CBF



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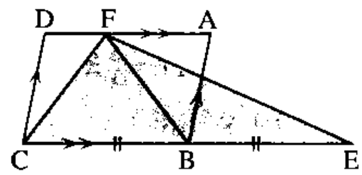
4. 📖 **In the opposite figure :**

ABCD is a parallelogram. E ∈ $\overrightarrow{CB}$ where BC = BE

Prove that : The area of Δ FEC = the area of ▱ ABCD



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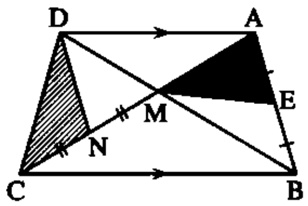
In the opposite figure :



ABCD is a quadrilateral whose diagonals intersect at M,

$\overline{AD} \parallel \overline{BC}$ and E is the midpoint of $\overline{AB}$,

N is the midpoint of $\overline{MC}$

Prove that : The area of Δ AEM = the area of Δ DNC

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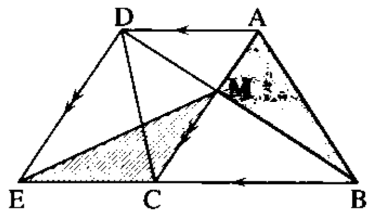
In the opposite figure :



$\overleftrightarrow{AD} \parallel \overleftrightarrow{BC}$, E $\in \overleftrightarrow{BC}$ and $\overleftrightarrow{AC} \parallel \overleftrightarrow{DE}$,

$\overline{AC} \cap \overline{BD} = \{M\}$

Prove that :

1. The area of Δ ABM = the area of Δ DCM = the area of Δ EMC
2. The area of Δ DBC = the area of Δ EBM

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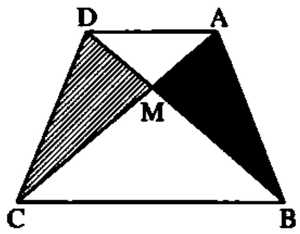
7. **In the opposite figure :**



ABCD is a quadrilateral , its diagonals intersect at M

and the area of Δ ABM = the area of Δ DCM

Prove that : $\overline{AD} \,//\, \overline{BC}$

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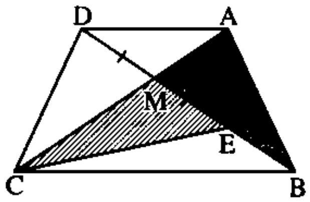
8. **In the opposite figure :**



ABCD is a quadrilateral whose diagonals are intersecting at M

and E $\in \overline{BM}$ where ME = MD

The area of Δ AMB = the area of Δ CME

Prove that : $\overline{AD} \,//\, \overline{BC}$

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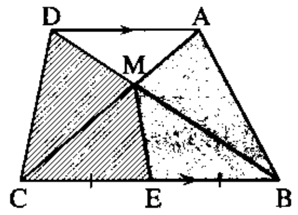
Homework

1. **In the opposite figure :**



$\overline{AD} \parallel \overline{BC}$, $\overline{AC} \cap \overline{BD} = \{M\}$,

E is the midpoint of $\overline{BC}$

Prove that : The area of the figure ABEM = the area of the figure DMEC

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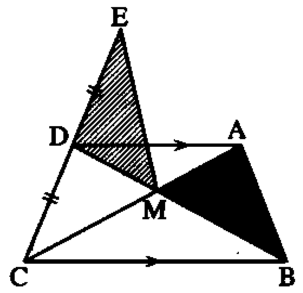
2. **In the opposite figure :**



$\overline{AD} \parallel \overline{BC}$ and $\overline{AC} \cap \overline{BD} = \{M\}$,

D is the midpoint of $\overline{EC}$

Prove that :

The area of Δ MDE = the area of Δ AMB

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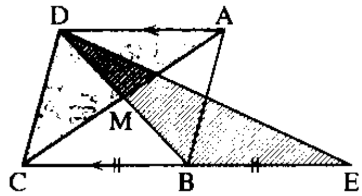
3. In the opposite figure :

ABCD is a parallelogram. Its diagonals intersect at M in which $\overline{AD}$ // $\overline{BC}$ and B is the midpoint of $\overline{EC}$

Prove that : The area of Δ EBD = the area of Δ ACD



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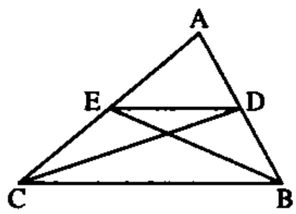
4. **In the opposite figure :**

ABC is a triangle in which D ∈ $\overline{AB}$ and E ∈ $\overline{AC}$ such that the area of Δ ABE = the area of Δ ACD

Prove that : $\overline{DE}$ // $\overline{BC}$



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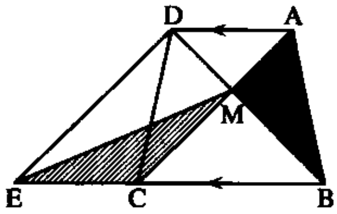
In the opposite figure :



ABCD is a quadrilateral in which $\overline{AD} \parallel \overline{BC}$

, $E \in \overrightarrow{BC}$ and $\overline{AC} \cap \overline{BD} = \{M\}$

The area of Δ ABM = the area of Δ ECM

Prove that : $\overline{DE} \parallel \overleftrightarrow{AC}$

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Lesson (16)
Areas of some geometric figures

The area of the rhombus = L × h where L is the side length and h is the height.

The area of the rhombus = $\frac{1}{2}$ of the product of the lengths of its two diagonals.

If the two legs of the trapezium are equal in length, then it is called an isosceles trapezium.
The following are the properties of the isosceles trapezium :

The two base angles of the isosceles trapezium are equal in measure.

The two diagonals of the isosceles trapezium are equal in length.

The isosceles trapezium has only one axis of symmetry which is the perpendicular bisector of its bases.

The area of the trapezium = half of the sum of lengths of the two parallel bases × height

The area of the trapezium = the length of the middle base × height

Complete each of the following :

1.	The area of rhombus whose perimeter is 20 cm. and height 4 cm. = ...
2.	The length of the diagonal of a square of area 50 cm^2 equals .... cm.
3.	The length of side of a square whose area equals the area of a rectangle with dimensions 9 cm. , 16 cm. = ...
4.	The length of the middle base of a trapezium whose area = 30 cm^2 and height 5 cm. equals ....

Homework

1.	The area of the rhombus = the side length × .... = $\frac{1}{2}$ of the product of ....
2.	The area of the square = the square of the length of .... = $\frac{1}{2}$ ....
3.	The length of the middle base of the trapezium equals ....
4.	The area of the trapezium = half of the sum of lengths of the two parallel bases × .... = the length of .... × its height
5.	The base angles of the isosceles trapezium are ....

6. The diagonals of an isosceles trapezium are ....

Choose the correct answer :

1. If the area of a square is 50 cm^2, then the length of its diagonal = ...

(a) 25 cm. (b) 5 cm. (c) 10 cm. (d) 20 cm.

2. If the perimeter of a rhombus is 24 cm. and its area = 30 $cm.^2$ then its height = ...

(a) 4 cm. (b) 5 cm. (c) 6 cm. (d) 12 cm.

3. If the product of the lengths of the diagonals of a rhombus = 96 cm^2 and its height is 6 cm., then its side length = ...

(a) 12 cm. (b) 8 cm. (c) 6 cm. (d) 4 cm.

4. If the area of a trapezium is 32 cm^2 and its height is 4 cm., then the length of its middle base = ...

(a) 4 cm. (b) 8 cm. (c) 14 cm. (d) 16 cm.

5. The trapezium in which the length of one of its parallel bases is 15 cm., and its area is 108 cm^2 and its height is 8 cm., then the length of the other base is ....

(a) 15 cm. (b) 4 cm. (c) 12 cm. (d) 27 cm.

6. The trapezium whose middle base length is x cm. and its height = $\frac{1}{2}$ the length of the middle base, its area = ... cm^2

(a) x^2 (b) $\frac{x^2}{2}$ (c) $\frac{x^2}{4}$ (d) $\frac{x^2}{8}$

Homework

1. The area of rhombus is 20 cm^2, the length of one of its diagonals is 5 cm., then the length of the other diagonal = ...

(a) 8 cm. (b) 4 cm. (c) 10 cm. (d) 15 cm.

2. The area of the square whose side length is 6 cm. .... the area of the square whose diagonal length is 8 cm.

(a) > (b) < (c) = (d) ≡

3. The trapezium in which the lengths of its two parallel bases are 15 cm. and 11 cm. Its middle base is with length ....

(a) 26 cm. (b) 15 cm. (c) 13 cm. (d) 11 cm.

4. If the area of the trapezium is 450 cm^2, and the lengths of its two parallel bases are 24 cm. and 12 cm. , then its height = ...

(a) 12.5 cm. (b) 25 cm. (c) 36 cm. (d) 52 cm.

Find the area of the following figures:

1. A rhombus of side length 6 cm. and its height = 5 cm. « 30 cm^2 »

....

2. A rhombus whose diagonal lengths are 24 cm. and 10 cm. « 120 cm^2 »

....

3. A square whose diagonal length = 10 cm. « 50 cm^2 »

....

4. A trapezium whose bases lengths are 8 cm. and 10 cm. and its height = 5 cm. « 45 cm^2 »

....

5. A trapezium whose middle base length is 7 cm. and its height = 6 cm. « 42 cm^2 »

....

Homework

1. A rhombus whose side length 12 cm. and its height = 8 cm. « 96 cm^2 »

....

2. A rhombus whose diagonals lengths are 8 cm. and 10 cm. « 40 cm^2 »

....

3. A square whose diagonal length = 8 cm. « 32 cm^2 »

....

4. A trapezium whose bases lengths are 6 cm. and 8 cm. and its height = 12 cm. « 84 cm^2 »

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5. A trapezium whose middle base length is 12 cm. and its height = 8 cm. « 96 cm^2 »

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Essay problems:

1. A square whose area equals the area of the rectangle whose dimensions are 2 cm. and 9 cm. Find the length of its diagonal. « 6 cm. »

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2. 🕮 Two pieces of land have equal areas , one of them has the shape of a rhombus whose diagonals are 18 m. and 24 m. , and the other one has the shape of a trapezium whose height is 12 m. Find the length of its middle base. « 18 m. »

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3. 🕮 The area of a trapezium is 180 cm^2 and its height is 12 cm. Find the lengths of its parallel bases if the ratio between their lengths is 3 : 2 « 18 cm. , 12 cm. »

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Homework

1. Two land pieces are equal in area , the first is in the shape of a square and the second is in the shape of a rhombus whose diagonals lengths are 8 metres and 16 metres. Find the perimeter of the square-shaped piece. « 32 cm. »

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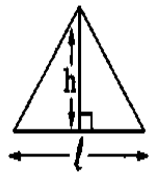
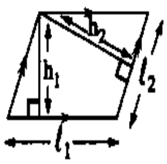
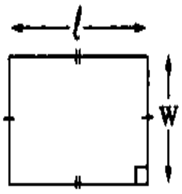
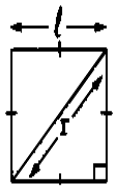
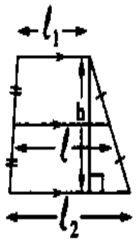
2. 🕮 Find the area of the rhombus whose perimeter is 52 cm. and the length of one of its diagonals is 10 cm. « 120 cm^2 »

The figure		The perimeter	The area
The triangle		The sum of the lengths of its three sides	$\frac{1}{2}$ of the base length × height $= \frac{1}{2}\, \ell \times h$
The parallelogram		The sum of lengths of two adjacent sides × 2 $= 2(\ell_1 + \ell_2)$	The base length × height $= \ell_1 \times h_1 = \ell_2 \times h_2$
The rectangle		2 (Length + Width) $= 2(\ell + w)$	Length × Width $= \ell \times w$
The square		Side length × 4 $= 4\ell$	Square of side length $= \ell^2$ or $\frac{1}{2}$ of the square of its diagonal length $= \frac{1}{2}\, r^2$
The rhombus		Side length × 4 $= 4\ell$	Side length × height $= \ell \times h$ or $\frac{1}{2}$ the product of the lengths of the two diagonals $= \frac{1}{2}\, r_1 \times r_2$
The trapezium		The sum of lengths of its sides	$\frac{1}{2}$ the sum of lengths of the two parallel bases × height $= \frac{1}{2}(\ell_1 + \ell_2) \times h$ or the length of the middle base × height $= \ell \times h$

Lesson (17)
Similarity

It is said that the two polygons P_1 and P_2 (of the same number of sides) are similar if the following two conditions are verified together :

1. Their corresponding angles are equal in measure.
2. The corresponding side lengths are proportional.

In this case , we write the polygon $P_1 \sim$ the polygon P_2

That means the polygon P_1 is similar to the polygon P_2

Remark (1)

In the two similar polygons P_1 and P_2 , the constant ratio among the lengths of the corresponding sides of P_1 and P_2 is called the ratio of enlargement or the drawing scale.

If the constant ratio is :

- Greater than 1 , then the polygon P_1 is an enlargement to the polgyon P_2
- Less than 1 , then the polygon P_1 is a minimizing of the polgyon P_2
- Equal to 1 , then the polygon P_1 is congruent to the polgyon P_2

Remark (2)

In order that two polygons are similar , the two conditions should be verified together and verifying one of them only is not enough to be similar.

Remark (3)

The congruent polygons are similar but it is not necessary that the similar polygons are congruent.

Remark (4)

All regular polygons of the same number of sides are similar.

Remark (5)

If each of two polygons is similar to a third polygon , then they are similar.

Remark (6)

The order of corresponding vertices should be kept in giving names of similar polygons that to help us finding the proportional sides lengths and the equal angles in measures.

i.e.

The ratio between the perimeters of two similar polygons = the ratio between the lengths of two corresponding sides.

A geometric fact :

The two triangles are similar if one of the two following conditions is verified :

1. The measures of their corresponding angles are equal.
2. The lengths of their corresponding sides are proportional.

Remarks

1. **The two right-angled triangles are similar if the measure of an acute angle in one of them is equal to the measure of an acute angle in the other.**
2. **The two equilateral triangles are similar.**
3. **The two isosceles triangles are similar if the measure of an angle in one of them equals the measure of the corresponding angle in the other.**

Complete each of the following :

1. If the measures of the corresponding angles in the two triangles are equal , then the two triangles are ....
2. If we have two polygons , their corresponding angles are .... and their corresponding sides lengths are .... , then the two polygons are similar.
3. If the ratio between the lengths of two corresponding sides in two similar triangles is equal to 1 , then the two triangles are ....
4. If two polygons are similar and the ratio between the lengths of two corresponding sides is 3 : 4 , then the ratio between their perimeters is ....

Homework

1. If two polygons are similar , then the corresponding .... are equal in measure.
2. If two polygons are similar , then the corresponding .... are proportional.
3. If each of two polygons is similar to a third , then they are ....
4. The two triangles are similar if the corresponding .... are proportional.

Choose the correct answer :

1. If the ratio between the lengths of two corresponding sides of two squares is 1 and the perimeter of one of them is 20 cm. , then the area of the other square = ...

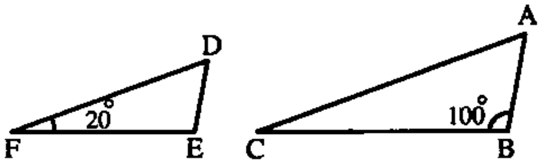
(a) 20 cm^2. (b) 25 cm^2. (c) 16 cm^2. (d) 25 cm.

2. **In the opposite figure :**

If $\Delta ABC \sim \Delta DEF$, then $m(\angle A) =$ ....

(a) 20° (b) 60°

(c) 80° (d) 100°



3. **In the opposite figure :**

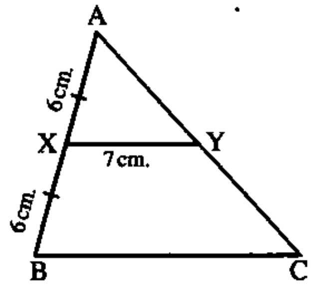
If $\Delta ABC \sim \Delta AXY$,

AX = XB = 6 cm.

XY = 7 cm. , then BC = ...

(a) 6 cm. (b) 7 cm.

(c) 12 cm. (d) 14 cm.

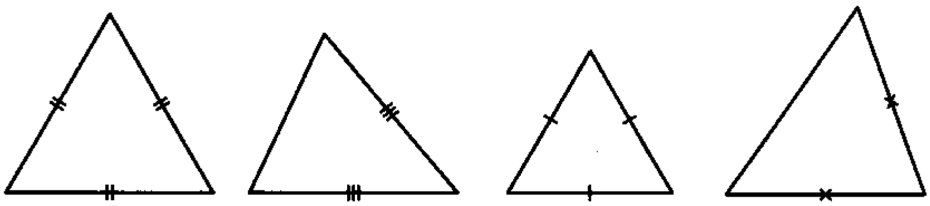


Homework

1. If $\Delta ABC \sim \Delta DEF$ and $AB = \frac{1}{5}$ DE , then perimeter of ΔABC = ... perimeter of ΔDEF

(a) 5 (b) 1 (c) $\frac{1}{5}$ (d) $\frac{2}{5}$

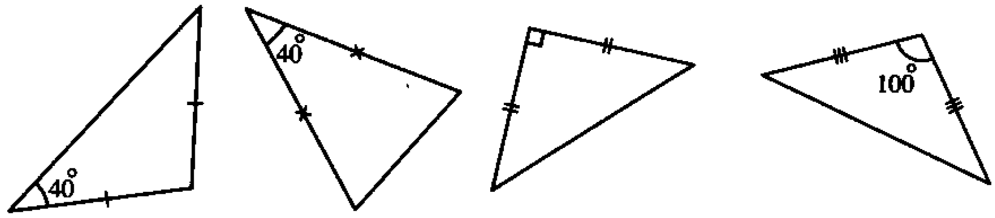
2. In the following figures , there are two similar triangles , they are ....



(1) (2) (3) (4)

(a) 1 , 2 (b) 1 , 3 (c) 1 , 4 (d) 2 , 4

3. In the following figures , there are two similar triangles , they are ....



(1) (2) (3) (4)

(a) 1 , 2 (b) 1 , 3 (c) 2 , 4 (d) 1 , 4

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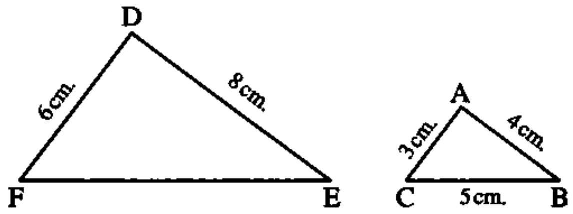
4. **In the opposite figure :**

If Δ ABC ~ Δ DEF , then EF = ...

(a) 5 cm. (b) 6 cm.

(c) 8 cm. (d) 10 cm.

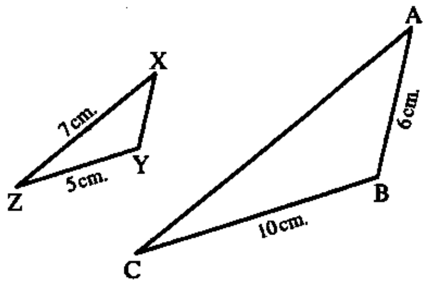


Essay problems:

1. **In the opposite figure :**

Δ ABC ~ Δ XYZ

Find : AC and XY « 14 cm. , 3 cm. »



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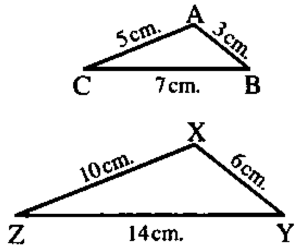
2. **In the opposite figure :**



1 **Prove that :** Δ ABC and Δ XYZ are similar.

2 If : m ($\angle$ B) + m ($\angle$ C) = 60° ,

find : m ($\angle$ X) « 120°»

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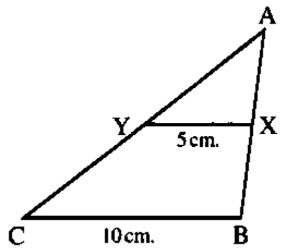
3. **In the opposite figure :**



If Δ AXY ~ Δ ABC

XY = 5 cm. and BC = 10 cm. ,

Prove that : **1** $\overline{XY}$ // $\overline{BC}$ **2** Y is the midpoint of $\overline{AC}$

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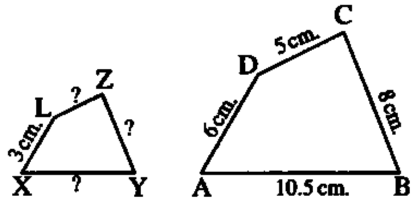
4. **In the opposite figure :**

The polygon ABCD ~ the polygon XYZL

If AB = 10.5 cm. , BC = 8 cm. , CD = 5 cm. ,

DA = 6 cm. and LX = 3 cm.

Find the length of each of : $\overline{XY}$, $\overline{YZ}$ and $\overline{ZL}$



« 5.25 cm. , 4 cm. , 2.5 cm. »

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5. **In the opposite figure :**

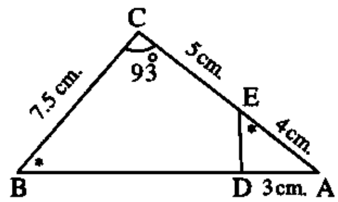
Δ ABC , D $\in \overline{AB}$, E $\in \overline{AC}$

, AE = 4 cm. , EC = 5 cm. , BC = 7.5 cm.

, AD = 3 cm. , m ($\angle$ AED) = m ($\angle$ B) and m ($\angle$ C) = 93°

1 **Prove that :** Δ AED ~ Δ ABC

2 **Find the length of each of :** $\overline{BD}$ and m ($\angle$ ADE)



« 9 cm. , 93° »

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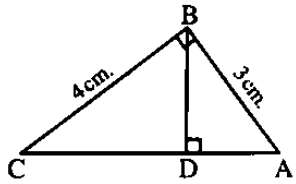
6. 📖 **In the opposite figure :**

ABC is a right-angled triangle at B in which :

AB = 3 cm. , BC = 4 cm. and $\overline{BD} \perp \overline{AC}$



1 Prove that : Δ BAC ~ Δ DAB

2 Find the length of each of : $\overline{AD}$ and $\overline{DC}$ « 1.8 cm. , 3.2 cm. »

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7. 📖 Two similar triangles, one of them has a perimeter of 74 cm. and the sides lengths of the other are 4.5 cm. , 6 cm. and 8 cm.

Find the length of the longest side in the first triangle. « 32 cm. »

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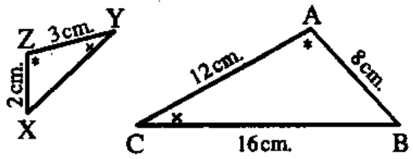
Homework

1. **Using the shown data in the figure , then prove that :**

Δ XYZ and Δ BCA are similar , then find the perimeter of Δ XYZ « 9 cm. »



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2. **In the opposite figure :**

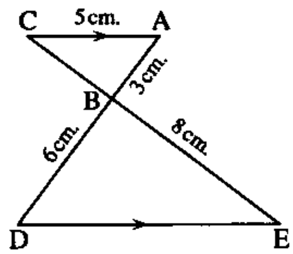
$\overline{AC} \parallel \overline{ED}$, $\overline{AD} \cap \overline{CE} = \{B\}$

, AC = 5 cm. , BE = 8 cm. , AB = 3 cm. and BD = 6 cm.

1 **Prove that :** Δ ABC $\sim$ Δ DBE

2 **Find the length of each of :** $\overline{BC}$ and $\overline{ED}$

3 **Find :** the ratio of enlargement.



« 4 cm. , 10 cm. , 2 »

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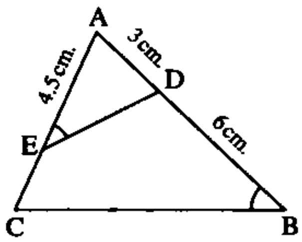
3. **In the opposite figure :**

m (∠ AED) = m (∠ B) , AD = 3 cm.

AE = 4.5 cm. and BD = 6 cm.

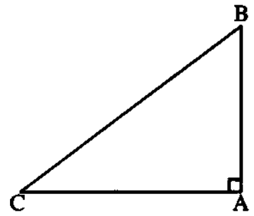


1 **Prove that : Δ ADE ~ Δ ACB**

2 **Find the length of : $\overline{EC}$** « 1.5 cm. »

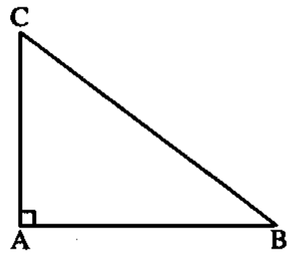
Lesson (18)
The converse of Pythagoras' theorem

We studied Pythagoras' theorem last year.
In the following , we will remind you of what you have studied.
If ABC is a right-angled triangle at A , then $\boxed{(BC)^2 = (AB)^2 + (AC)^2}$
Now we shall study the converse of Pythagoras' theorem.



In a triangle , if the sum of the areas of two squares on two sides is equal to the area of the square on the third side , then the angle opposite to this side is a right angle.

In Δ ABC , if :
$(AB)^2 + (AC)^2 = (BC)^2$,
then : m ($\angle$ A) = 90°



We can state this theorem as follows :

In a triangle , if the square of the length of a side is equal to the sum of the squares of the lengths of the other two sides , then the angle opposite to this side is a right angle.

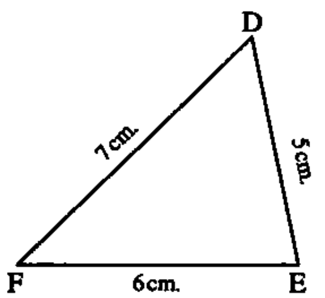
Corollary

In Δ ABC , if $\overline{AC}$ is the longest side and if $(AC)^2 \neq (AB)^2 + (BC)^2$, then m ($\angle$ B) $\neq$ 90° and the triangle is not right-angled.

Complete each of the following :

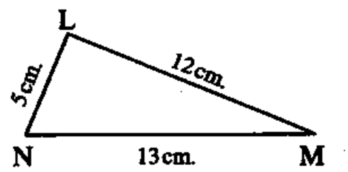
Complete and show which of the following triangles is a right-angled triangle :

1



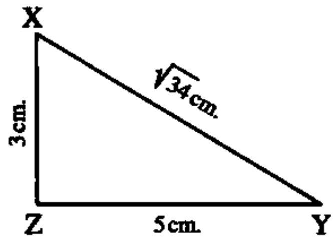
$(DF)^2 =$ ....
$(DE)^2 + (EF)^2 =$ ....
$\therefore$ The triangle is ....

2



$(MN)^2 =$ ....
$(ML)^2 + (NL)^2 =$ ....
$\therefore$ The triangle is ....

3

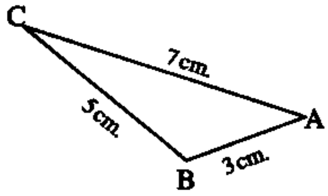


$(XY)^2 = (\sqrt{34})^2 = \cdots\cdots\cdots$

$(YZ)^2 + (ZX)^2 = \cdots\cdots\cdots$

$\therefore$ The triangle is ·········

4



$(AC)^2 = \cdots\cdots\cdots$

$(AB)^2 + (BC)^2 = \cdots\cdots\cdots$

$\therefore$ The triangle is ·········

Homework

In each of the following figures

Prove that : m ($\angle$ B) = 90°

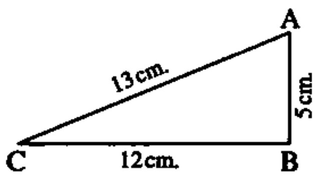


Fig. (1)

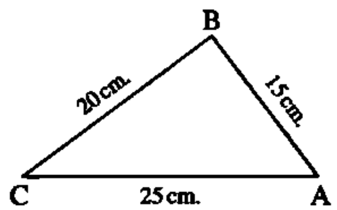


Fig. (2)

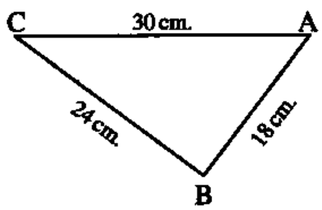


Fig. (3)

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Essay problems:

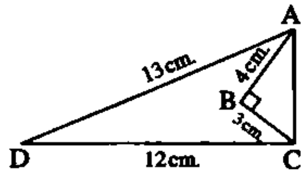
1. **In the opposite figure :**

m (∠ B) = 90° , AB = 4 cm. , BC = 3 cm.

AD = 13 cm. and DC = 12 cm.

Find : The area of the figure ABCD

« 24 cm^2 »



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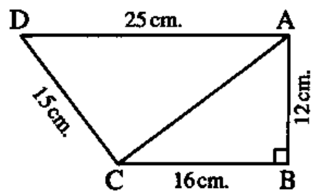
2. **In the opposite figure :**



ABCD is a quadrilateral in which : m (∠ B) = 90° ,

AB = 12 cm. , BC = 16 cm. , CD = 15 cm. and DA = 25 cm.

1 **Find :** The length of $\overline{AC}$

2 **Prove that :** m (∠ ACD) = 90°

« 20 cm. »

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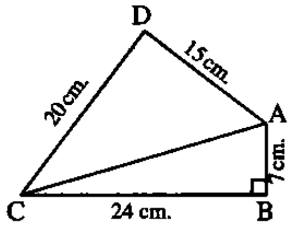
Homework

1 **In the opposite figure :**



ABCD is a quadrilateral in which : m (∠ ABC) = 90° ,

AB = 7 cm. , BC = 24 cm. , CD = 20 cm. and DA = 15 cm.

Prove that : m (∠ ADC) = 90°

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2. ABC is a triangle in which : AB = 4.5 cm. , BC = 7.5 cm. , AC = 6 cm.

Prove that : Δ ABC is right-angled.

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3.

In the opposite figure :

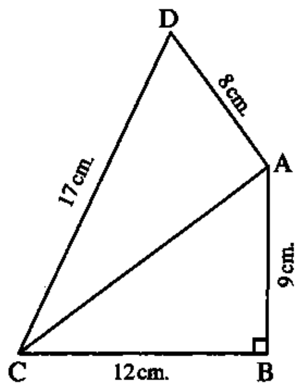
ABCD is a quadrilateral in which : m (∠ B) = 90° ,

AB = 9 cm. , BC = 12 cm. ,

CD = 17 cm. and DA = 8 cm.

Prove that : m (∠ DAC) = 90° ,

then find : The area of the figure ABCD « 114 cm^2 »



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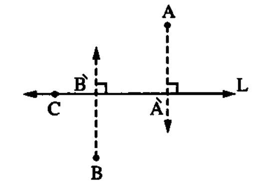
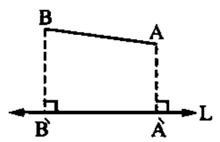
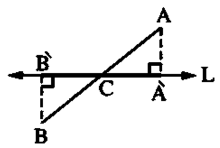
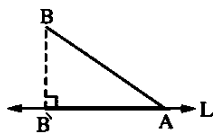
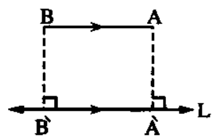
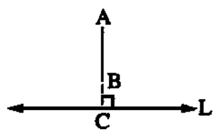
Lesson (19)
Projections

The projection of a point on a straight line:

1 The projection of a point on a straight line is the point of intersection of the perpendicular segment from this point and the straight line. **2** If the point lies on the straight line, its projection on it is the same point.	

The projection of a line segment on a straight line:

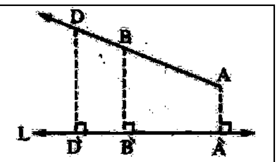
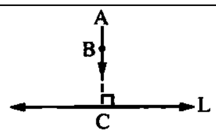
The projection of a line segment on a given straight line is the line segment whose two endpoints are the projections of the two endpoints of the main line segment on this straight line.

				
$A'B' < AB$	$A'B' < AB$	$AB' < AB$	$A'B' = AB$	$A'B' =$ zero

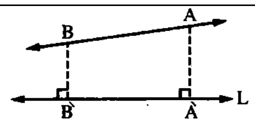
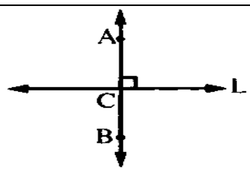
From the table, we notice that :

The length of the projection of a line segment on a given straight line $\leq$ the length of the line segment.

The projection of a ray on a straight line:

The projection of a ray on a straight line not perpendicular to it is a ray $\subset$ this straight line.	
The projection of a ray on a straight line perpendicular to it is a point belonging to the straight line.	

The projection of a straight line on another straight line:

The projection of a straight line on a straight line not perpendicular to it is a straight line.	
The projection of a straight line on a straight line perpendicular to it is the point of intersection of the two straight lines.	

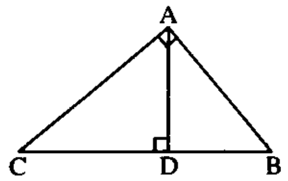
Complete each of the following :

1.

In the opposite figure :

Δ ABC is right-angled at A and $\overline{AD} \perp \overline{BC}$

Complete the following :



1. The projection of $\overline{AB}$ on $\overleftrightarrow{BC}$ is ....
2. The projection of $\overline{AC}$ on $\overleftrightarrow{BC}$ is ....
3. The projection of $\overline{BC}$ on $\overleftrightarrow{AC}$ is ....
4. The projection of $\overline{BC}$ on $\overleftrightarrow{AB}$ is ....
5. The projection of $\overline{AC}$ on $\overleftrightarrow{AD}$ is ....
6. The projection of $\overline{AD}$ on $\overleftrightarrow{BC}$ is ....
7. The projection of $\overline{AB}$ on $\overleftrightarrow{AD}$ is ....

2. If $X \in \overleftrightarrow{AB}$, then the projection of X on $\overleftrightarrow{AB}$ is ....

3. If $\overline{AB} \perp \overline{BC}$, then the projection of $\overline{AB}$ on $\overleftrightarrow{BC}$ is ....

4. In Δ ABC , if m ($\angle$B) = 90° , then the projection of C on $\overleftrightarrow{AB}$ is ....

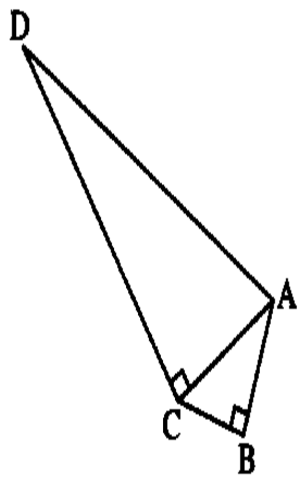
5. ABC is a right-angled triangle at A , then the projection of $\overline{BA}$ on $\overleftrightarrow{AC}$ is ....

Homework

1. In the opposite figure :

m($\angle$ B) = m ($\angle$ ACD) = 90°

Complete :



1. The projection of $\overline{AD}$ on $\overleftrightarrow{CD}$ is ....
2. The projection of $\overline{AC}$ on $\overleftrightarrow{CD}$ is ....
3. The projection of $\overline{AC}$ on $\overleftrightarrow{AB}$ is ....

Choose the correct answer :

1. The projection of a ray on a straight line not perpendicular to it is ....
(a) a point. (b) a line segment. (c) a ray. (d) a straight line.

2. The length of the projection of a line segment on a given straight line .... the length of the line segment itself.
(a) $\leq$ (b) $>$ (c) $\geq$ (d) $=$

3. The length of the projection of a line segment on a straight line parallel to it .... the length of the main line segment.
(a) $<$ (b) $>$ (c) $=$ (d) $\neq$

4. The length of the projection of a line segment on a straight line perpendicular to it is ....
(a) greater than the length of the main line segment.
(b) equal to the length of the main line segment.
(c) greater than or equal to the length of the main line segment.
(d) equal to zero.

Homework .

1. The projection of a point on a given straight line is ....
(a) a point. (b) a line segment. (c) a ray. (d) a straight line.

2. The projection of a line segment on a straight line not perpendicular to it is ....
(a) a ray. (b) a point. (c) a line segment. (d) a straight line.

3. The projection of a line segment on a straight line perpendicular to it is ....
(a) a point. (b) a line segment. (c) a ray. (d) a straight line.

Essay problems:

1. **In the opposite figure :**

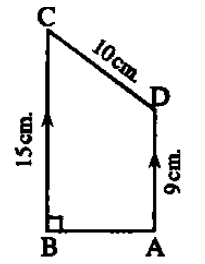
ABCD is a trapezium in which $\overline{AD} \parallel \overline{BC}$ and $m(\angle ABC) = 90°$

If AD = 9 cm. , DC = 10 cm. and CB = 15 cm.

Find :

1 The length of the projection of $\overline{DC}$ on $\overleftrightarrow{BC}$

2 The length of the projection of $\overline{DC}$ on $\overleftrightarrow{AB}$



« 6 cm. , 8 cm. »

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Homework

1. **In the opposite figure :**

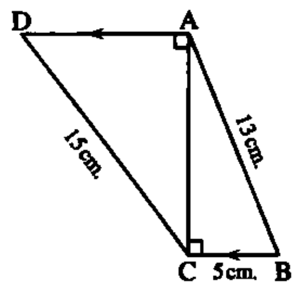
$\overline{AD} \parallel \overline{BC}$, AB = 13 cm. , BC = 5 cm. ,

CD = 15 cm. and $m(\angle ACB) = m(\angle DAC) = 90°$

Find :

1 The length of the projection of $\overline{AB}$ on $\overleftrightarrow{AC}$

2 The length of the projection of $\overline{CD}$ on $\overleftrightarrow{AD}$



« 12 cm. , 9 cm. »

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Lesson (20)
Euclidean Theorem

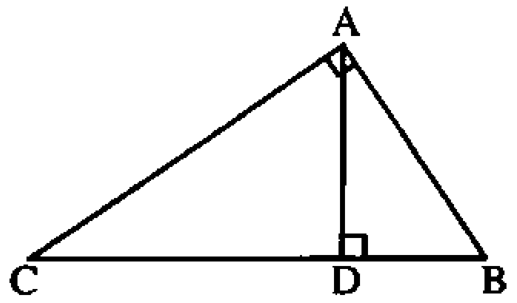
In the right-angled triangle , the area of the square on a side of the right angle is equal to the area of the rectangle whose dimensions are the length of the projection of this side on the hypotenuse and the length of the hypotenuse.

$\boxed{(AB)^2 = DB \times BC}$

$\boxed{(AC)^2 = DC \times BC}$

$\boxed{(DA)^2 = DB \times DC}$

$\boxed{DA = \dfrac{BA \times AC}{BC}}$



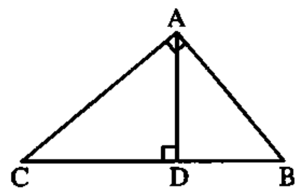
Complete each of the following :

1.

In the opposite figure :

Δ ABC is right-angled at A , $\overline{AD} \perp \overline{BC}$

Complete each of the following :



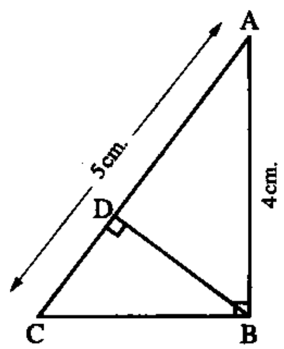
1. $(AC)^2 = \cdots\cdots + \cdots\cdots$
2. $(AC)^2 = \cdots\cdots - \cdots\cdots$
3. $(AC)^2 = \cdots\cdots \times \cdots\cdots$
4. $(AD)^2 = \cdots\cdots \times \cdots\cdots$
5. $AC \times AB = \cdots\cdots \times \cdots\cdots$
6. $\Delta ABC \sim \Delta \cdots\cdots \sim \Delta \cdots\cdots$

Homework

1. **In the opposite figure :**

ABC is a triangle in which m ($\angle$ ABC) = 90° , AB = 4 cm. , AC = 5 cm. and $\overline{BD} \perp \overline{AC}$

Complete :



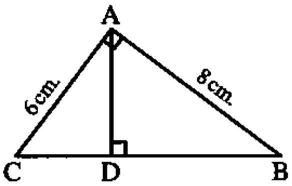
1. BC = ········ cm.
2. AD = ········ cm.
3. BD = ········ cm.
4. The area of Δ DBC = ········ cm^2

Essay problems:

1. **In the opposite figure :**

ABC is a triangle in which m (∠ BAC) = 90° , $\overline{AD} \perp \overline{BC}$, AB = 8 cm. and AC = 6 cm.

Find : BD , CD and AD



« 6.4 cm. , 3.6 cm. , 4.8 cm. »

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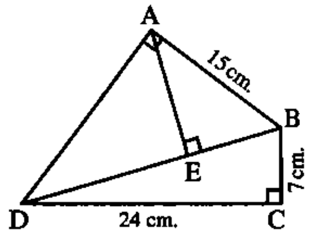
2. **In the opposite figure :**

ABCD is a quadrilateral where

m (∠ BCD) = m (∠ BAD) = 90° ,

$\overline{AE} \perp \overline{BD}$, BC = 7 cm. , CD = 24 cm.

and AB = 15 cm.



Find : 1 The length of each of $\overline{BD}$ and $\overline{AD}$

2 The length of the projection of $\overline{AB}$ on $\overleftrightarrow{BD}$

3 The length of the projection of $\overline{AD}$ on $\overleftrightarrow{AE}$

« 25 cm. , 20 cm. , 9 cm. , 12 cm. »

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3. **In the opposite figure :**

Δ ABC is right-angled at B

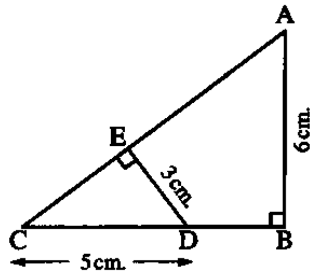
, $\overline{DE} \perp \overline{AC}$, AB = 6 cm.

, ED = 3 cm. and CD = 5 cm.

Prove that : Δ CED ~ Δ CBA

and find : The length of $\overline{AC}$

and the length of the projection of $\overline{AB}$ on $\overleftrightarrow{AC}$



« 10 cm. , 3.6 cm. »

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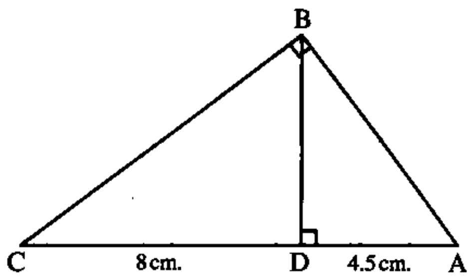
Homework

1. **In the opposite figure :**

Δ ABC is right-angled at B and $\overline{BD} \perp \overline{AC}$

If AD = 4.5 cm. and DC = 8 cm. ,

find : The length of each of $\overline{AB}$, $\overline{BC}$ and $\overline{BD}$



« 7.5 cm. , 10 cm. , 6 cm. »

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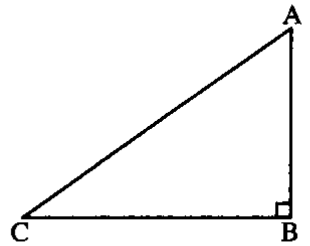
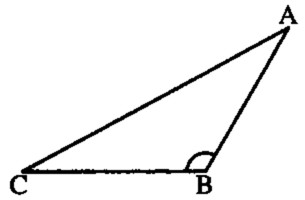
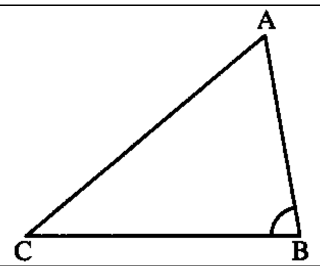
Lesson (21)
Classifying triangles according to their angles

If $(AC)^2 = (AB)^2 + (BC)^2$, then m ($\angle$ ABC) = 90° and ABC is a right-angled triangle.	
If $(AC)^2 > (AB)^2 + (BC)^2$, then m ($\angle$ ABC) > 90° and ABC is an obtuse-angled triangle.	
If $(AC)^2 < (AB)^2 + (BC)^2$, then m ($\angle$ ABC) < 90° and ABC is an acute-angled triangle.	

Complete each of the following :

1. In Δ ABC , if $(AB)^2 = (AC)^2 - (BC)^2$, then $\angle$ C is ……….
2. In Δ ABC , if $(AC)^2 - (AB)^2 = (BC)^2 - 3$, then $\angle$ B is ……….
3. In Δ ABC , if $(AB)^2 + (BC)^2 = 48$ cm^2 , AC = 7 cm. , then $\angle$ B is ……….
4. In Δ XYZ , if 90° < m ($\angle$ Y) < 180° , then $(XZ)^2$ ………. $(XY)^2 + (YZ)^2$
5. If $\angle$ A complements $\angle$ B in Δ ABC , then $(AB)^2$ ………. $(AC)^2 + (BC)^2$
6. If the two lengths of two sides in a triangle are 3 cm. and 5 cm , then the length of the third side is between ………. , ……….
7. ABC is a triangle whose sides lengths are 6 cm., 8 cm. and 11 cm. Δ ABC is similar to the triangle XYZ , then Δ XYZ is ………. according to its angles.
8. In Δ XYZ , if (XZ – XY) (XZ + XY) < $(ZY)^2$, then $\angle$ Y is ……….

Homework

1. In Δ ABC , if $(AB)^2 = (BC)^2 + (AC)^2$, then : m ($\angle$ ……….) = 90°
2. In Δ ABC , if $(AB)^2 < (AC)^2 + (BC)^2$, then $\angle$ C is ……….

3. In Δ ABC , if $(AB)^2 + (BC)^2 < (AC)^2$, then $\angle$ B is ....

4. In Δ XYZ , if $(XY)^2 = (YZ)^2 + (ZX)^2$, then $\angle$ Z is ....

5. In Δ XYZ , if $(YZ)^2 > (XZ)^2 - (XY)^2$, then $\angle$ Y is ....

Choose the correct answer :

1. Atriangle whose side lengths are : 5 cm , 12 cm and 13 cm. its area = ... cm^2
(a) 30 (b) 32.5 (c) 78 (d) 60

2. ABC is an obtuse-angled triangle at A , if AB = 4 cm. , BC = 7 cm. , then AC can be equals .... cm.
(a) 5 (b) 6 (c) 7 (d) 8

3. ABC is a triangle in which : $(BC)^2 = (AB)^2 + (AC)^2$, m ($\angle$ B) = 40° , then m ($\angle$ C) = ...
(a) 40° (b) 50° (c) 90° (d) 140°

Homework .

1. ABC is an obtuse-angled triangle at B if AB = 5 cm. , BC = 3 cm. , then AC can be equals .... cm.
(a) 4 (b) 5 (c) 7 (d) 8

2. ABC is an acute-angled triangle in which AB = 6 cm. , BC = 8 cm. , then the length of $\overline{AC}$ can be equals .... cm.
(a) 2 (b) 6 (c) 10 (d) 14

Essay problems:

1. Identify the type of $\angle$ A in Δ ABC if AB = 6 cm. , BC = 10 cm. and AC = 8 cm.

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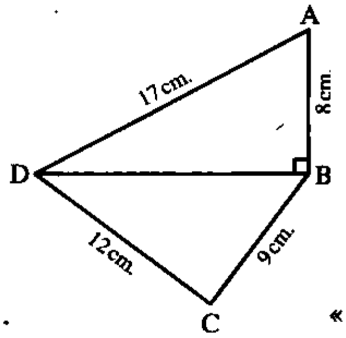
2. Identify the type of $\angle B$ in ΔABC if $AB = 10$ cm. , $BC = 12$ cm. and $AC = 15$ cm.

3. **In the opposite figure :**



ABCD is a quadrilateral in which $AB = 8$ cm. , $BC = 9$ cm. , $CD = 12$ cm. , $AD = 17$ cm. and $\overline{DB} \perp \overline{AB}$

1. Find the length of the projection of $\overline{AD}$ on $\overleftrightarrow{BD}$ « 15 cm. »
2. Determine the type of ΔBCD according to its angles.

Homework

1. Identify the type of $\angle Y$ in ΔXYZ if $XY = 4$ cm. , $YZ = 5$ cm. and $XZ = 7$ cm.

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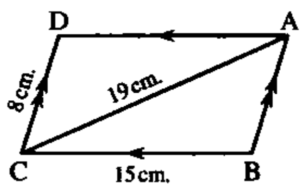
2. **In the opposite figure :**



ABCD is a parallelogram in which

$BC = 15$ cm. , $CD = 8$ cm. and $AC = 19$ cm.

Prove that : $\angle ABC$ is an obtuse angle.

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Best Wishes